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The mechanism of magnetic field origin at vortical processes in stellar plasma

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Abstract. Because of the ohmic loss in stellar plasma, the electric currents and the magnetic fields connected to these currents gradually decay. Mechanisms "dynamo"(Priest, 1985) are able to compensate attenuation and to strengthen magnetic fields. However these mechanisms do not operate if the magnetic field in a star did not exist initially or practically completely decayed. It is shown in the present work that the magnetic field develops from a "zero"-state in stationary vortical processes in stellar plasma. Parameters of arising magnetic field are estimated for a simplified plane model of a vortical process. A particular consideration is performed for the magnetic field of a sunspot.

1 Basic statements and assumptions

It is accepted that the substance in the process area is completely ionized hydrogen plasma. This plasma is actually a complex gas consisting of two components. These components are unitary charged gases of protons and electrons. Dynamics of such plasma is described by the system of the modified equations of Navier-Stokes separately for the proton gas (1) and the electron gas (2) (Matveyev, 1980; Lifshits, Pitayevsky, 2002; Loytsyansky, 2003):

$$\rho_i \frac{d\mathbf{v}_i}{dt} = \mathbf{f}_{pi} + \mathbf{f}_{Gi} + \mathbf{f}_{\eta i} + \mathbf{f}_{Ei} + \mathbf{f}_{Mi} + \boldsymbol{\tau}_{ei} \tag{1}$$

$$\rho_e \frac{d\mathbf{v}_e}{dt} = \mathbf{f}_{pe} + \mathbf{f}_{Ge} + \mathbf{f}_{\eta e} + \mathbf{f}_{Ee} + \mathbf{f}_{Me} + \boldsymbol{\tau}_{ie}, \tag{2}$$

where ρ_i, ρ_e is the density of the proton gas and the electron gas accordingly; $d\mathbf{v}_i/dt$, $d\mathbf{v}_e/dt$ are full derivatives of velocities; $\mathbf{f}_{pi}$, $\mathbf{f}_{Gi}$, $\mathbf{f}_{\eta i}$, $\mathbf{f}_{Ei}$, $\mathbf{f}_{Mi}$ and $\mathbf{f}_{pe}$, $\mathbf{f}_{Ge}$, $\mathbf{f}_{\eta e}$, $\mathbf{f}_{Ee}$, $\mathbf{f}_{Me}$ are the densities of the pressure force, the gravitational force, the friction force, the electric and magnetic forces respectively for the proton gas and the electron gas; $\boldsymbol{\tau}_{ei}$ is the density of the force with which the electron gas acts upon the proton gas; $\boldsymbol{\tau}_{ie}$ is the analogous density for the electron gas.

1.1 Density of gases

If it is not stipulated specially, we consider that the concentration of protons and electrons is the same: $n \equiv n_i \approx n_e$. The ratio of the densities of the proton gas $\rho_i = m_i n$ and the electron gas $\rho_i = m_i n$ is the same as the ratio of proton mass $m_i = 1.67{\cdot}10^{-27}$ kg and electron mass $m_i = 9.1{\cdot}10^{-31}$ kg:

$$\rho_i \rho_e = m_i / m_e = 1836. \tag{3}$$

The plasma density $\rho = \rho_i + \rho_e$ practically precisely equals the density of the proton gas: $\rho \approx \rho_i$. For the sake of simplicity we consider that plasma is incompressible and, moreover, that density and the concentration of particles are constant.

1.2 Speed of gases

Due to the big mutual friction, speeds of the proton and electron gases are very close to each other. In calculation of many parameters it is possible to consider that these speeds are equal to the speed of the plasma as a whole ($\mathbf{v} \approx \mathbf{v}_i \approx \mathbf{v}_e$). Thus,

$$\frac{d\mathbf{v}}{dt} = \frac{d\mathbf{v}_i}{dt} = \frac{d\mathbf{v}_e}{dt}, \quad \Delta\mathbf{v} = \Delta\mathbf{v}_i = \Delta\mathbf{v}_e. \tag{4}$$

1.3 Pressure of plasma

Because of the fast exchange of heat, the proton gas and the electron gas are in thermal balance and their temperature equals the temperature of the plasma as the whole $T = T_i = T_e$. The partial pressures of the proton gas and the electron gas $p_i = nk_BT$, $p_e = nk_BT$ (where $k_B = 1.38{\cdot}10^{-23}$ J/K is the Boltzmann constant) equal to a half of the plasma pressure $p_i = p_e = p/2$ under Dalton's law. The density of the pressure force of both the proton gas and the electron gas (Lifshits, Pitayevsky, 2002) equals:

$$\mathbf{f}_{pi} = \mathbf{f}_{pe} = -\operatorname{grad} p_i = -\operatorname{grad} p_e = -\operatorname{grad} p/2. \tag{5}$$

1.4 Gravitational force

The density of gravitational force acting on the proton gas is $\mathbf{f}_{Gi} = \rho_i \mathbf{g}$ and on the electron gas — $\mathbf{f}_{Ge} = \rho_e \mathbf{g}$, where $\mathbf{g}$ is the gravitational acceleration. This acceleration always has a potential function: $\mathbf{g} = -\operatorname{grad} G$. Thus,

$$\mathbf{f}_{Gi} = -\rho_i \operatorname{grad} G, \quad \mathbf{f}_{Ge} = -\rho_e \operatorname{grad} G. \tag{6}$$

1.5 Electric forces

The density of the total charge of the proton gas is $q_i = n_i e$ and of the electron gas is $q_i = n_e e$ (where $e = 1.6{\cdot}10^{-19}$ C is the elementary charge). The arising electric field hinders the further division of charges in the plasma. Therefore the difference of concentrations of protons and electrons is very small $|n_i - n_e| << n_i$ (Lifshits, Pitayevsky, 2002). Both the density of the total charge $q = q_i + q_e$ and the electric intensity $\mathbf{E}$ ($\operatorname{div} \mathbf{E} = q/\varepsilon_0$, $\varepsilon_0 = 8.85{\cdot}10^{-12}$ F/m is the electric constant) depend on this difference. The electrostatic field is potential. The electric forces acting on the unit volumes of the proton and electron gases in a field with potential φ, are:

$$\mathbf{f}_{Ei} = -\mathbf{f}_{Ee} = ne\,\mathbf{E} = -ne \operatorname{grad} \varphi. \tag{7}$$

1.6 Electric current and magnetic force

The density of an electric current depends on the speed difference between the gases of protons and electrons (Matveyev, 1980):

$$\mathbf{j} = ne(\mathbf{v}_i - \mathbf{v}_e). \tag{8}$$

The stationary current can be only solenoidal, as $\operatorname{div}\mathbf{j} = 0$. This current cannot be caused by an electric field as this field is potential.

According to the Lorentz's law, the electric intensity arising at the movement of a charge with a speed $\mathbf{v}$ in a magnetic field with induction $\mathbf{B}$ equals the vector product: $\mathbf{E}_M = [\mathbf{vB}]$. For the proton and electron gases, accordingly:

$$\mathbf{E}_{Mi} = [\mathbf{v}_i\mathbf{B}], \quad \mathbf{E}_{Me} = [\mathbf{v}_e\mathbf{B}].$$

The corresponding specific magnetic forces are written out as:

$$\mathbf{f}_{Mi} = ne\,[\mathbf{v}_i\mathbf{B}], \quad \mathbf{f}_{Me} = -ne\,[\mathbf{v}_e\mathbf{B}]. \tag{9}$$

The specific magnetic force acting on the plasma as a whole equals the sum:

$$\mathbf{f}_M = \mathbf{f}_{Mi} + \mathbf{f}_{Me} = ne\,[\mathbf{v}_i\mathbf{B}] - ne\,[\mathbf{v}_e\mathbf{B}] = [\mathbf{jB}]. \tag{10}$$

1.7 Viscosity of plasma and its components

Specific forces of friction in the gases of protons and electrons and in the plasma as a whole depend on the corresponding dynamic viscosities η_i, η_e, and $\eta = \eta_i + \eta_e$ (Kulikovsky, Lyubimov, 1962):

$$f_{\eta i} = \eta_i\,\Delta\mathbf{v}, \quad f_{\eta e} = \eta_e\,\Delta\mathbf{v}, \quad f_\eta = \eta\,\Delta\mathbf{v}. \tag{11}$$

The ratio of the dynamic viscosities of the gases of protons and electrons equals a square root from the ratio of masses of a proton and electron (Lifshits, Pitayevsky, 2002):

$$\eta_i/\eta_e = \sqrt{m_i/m_e} = 42.8 \tag{12}$$

All the specific viscosities are sharply increasing with temperature as $T^{5/2}$, however the ratio (12) is unaltered. This ratio is smaller than the ratio of densities of the proton gas and the electron gas. This will be used below.

1.8 Mutual viscosity of gases of protons and electrons

Let us assume that some extraneous electric field (Matveyev, 1980) with an intensity $\mathbf{E}$ is situated in the plasma. This field acts upon the gases of protons and electrons in opposite directions and causes their mutual movement. Thus the external electric force with a density $\mathbf{f}_{Ei} = ne\mathbf{E}$ acts upon the gas of protons. The force of the internal friction of the electron gas $\tau_{ei} = -\mathbf{f}_{Ei}$ compensates the external force. The forces acting upon the electron gas equal previous ones by the absolute magnitude

$$\mathbf{f}_{Ee} = -\mathbf{f}_{Ei} = -ne\mathbf{E}, \quad \tau_{ie} = -\mathbf{f}_{Ee} = \tau_{ei}.$$

The force of the mutual friction depend on the difference $\mathbf{v}_i - \mathbf{v}_e$ of average velocities of protons $\mathbf{v}_i$ and electrons $\mathbf{v}_e$ and factor η of the mutual friction (Lifshits E.M., Pitayevsky A.P., 2002):

$$\mathbf{f}_{Ei} = -\mathbf{f}_{Ee} = -\tau_{ei} = \tau_{ie} = \eta\left(\mathbf{v}_i - \mathbf{v}_e\right). \tag{13}$$

Taking into account (8) we write out (13)

$$\mathbf{f}_{Ei} = \frac{\eta}{ne}\mathbf{j}. \tag{14}$$

Using the relationship $\mathbf{f}_{Ei} = ne\mathbf{E}$, we obtain

$$\mathbf{j} = \frac{n^2e^2}{\eta}\mathbf{E}. \tag{15}$$

According to Ohm's law in the differential form, the multiplier before $\mathbf{E}$ in (15) is the plasma conductivity γ and hence

$$\gamma = n^2e^2/\eta, \quad \eta = n^2e^2/\gamma. \tag{16}$$

In (Priest, 1985) is given the following dependence of the plasma conductivity on the temperature

$$\gamma = 1.53 \cdot 10^{-2} \cdot T^{3/2}/L_e, \tag{17}$$

where L_e is the Coulomb logarithm.

With the temperature rise the conductivity increases, while the factor of the mutual friction η decreases. However in the whole range of temperatures the mutual viscosity is very great in comparison with the viscosity of gases of protons and electrons taken separately. Therefore the difference in the speed of the gases is less than the speed of the plasma as a whole $|\mathbf{v}_i - \mathbf{v}_e| < |\mathbf{v}|$.

2 Set of equations for stationary vortical process in plasma

We consider a stationary process in some volume of a star. We assume that external electromagnetic fields are absent. These estimations are used in the initial equations (1, 2):

$$\rho_i\frac{d\mathbf{v}}{dt} = -\mathrm{grad}(p/2) - ne\,\mathrm{grad}\,\varphi - \rho_i\,\mathrm{grad}\,G + \eta_i\Delta\mathbf{v} - \frac{ne}{\gamma}\mathbf{j} + ne\left[\mathbf{v}_i\mathbf{B}\right] \tag{18}$$

$$\rho_e\frac{d\mathbf{v}}{dt} = -\mathrm{grad}\,(p/2) + ne\,\mathrm{grad}\varphi - \rho_e\,\mathrm{grad}\,G + \eta_e\Delta\mathbf{v} + \frac{ne}{\gamma}\mathbf{j} - ne\left[\mathbf{v}_e\mathbf{B}\right] \tag{19}$$

It is necessary to add following equalities for stationary process:

$$\frac{\partial \mathbf{v}}{\partial t} = 0, \quad \frac{\partial \mathbf{B}}{\partial t} = 0, \quad \operatorname{div} \mathbf{j} = 0. \tag{20}$$

Besides, for the sake of simplicity, we believe that the plasma in the process without field is uncompressible, hence (Loytsyansky, 2003):

$$\operatorname{div} \mathbf{v} = 0. \tag{21}$$

Equations (18, 19) in the initial form are complex and inconvenient. Use of physical restrictions and initial conditions allows simplifying them. So summarizing (18, 19) and taking into account the previous dependences, we derive the known equation for the plasma as a whole (Kulikovsky, Lyubimov, 1962):

$$\rho \frac{d\mathbf{v}}{dt} = -\operatorname{grad} p - \rho \operatorname{grad} G + \eta \Delta \mathbf{v} + [\mathbf{j}\,\mathbf{B}]. \tag{22}$$

Here potential forces are mutually counterbalanced. Carrying out the curl operation in (22), we exclude these forces:

$$\rho \frac{d\Omega}{dt} = \eta \Delta \Omega + \operatorname{curl} [\mathbf{j}\,\mathbf{B}], \tag{23}$$

where $\Omega = \operatorname{curl} \mathbf{v}$ is the plasma vorticity; $\frac{d\Omega}{dt} = \frac{\partial \Omega}{\partial t} + (\mathbf{v}\nabla)\,\Omega$ is the full derivative of the vorticity.

Equation (23) contains only a part of the initial information of (18, 19). One more equation is obtained [6, 7] by means of exception of laplacian $\Delta \mathbf{v}$. For this purpose, we multiplied (18) by $\eta_e/\eta_i = \sqrt{\rho_e/\rho_i} = 1/42.8$ and then subtracted (19) from the equation obtained:

$$\begin{aligned} (\sqrt{\rho_i \rho_e} - \rho_i) \frac{d\mathbf{v}}{dt} &= (1 - \frac{\eta_e}{\eta_i}) \operatorname{grad}(p/2) - (1 + \frac{\eta_e}{\eta_i})\, ne \operatorname{grad} \varphi - \\ -\sqrt{\rho_i \rho_e} \operatorname{grad} G &- (1 + \frac{\eta_e}{\eta_i}) \frac{ne}{\gamma} \mathbf{j} + (1 + \frac{\eta_e}{\eta_i}) ne\, [\mathbf{v}\mathbf{B}]\,. \end{aligned} \tag{24}$$

Further in multipliers on the left side and on the right side we neglect addend $\eta_e/\eta_i = 1/42.8$ and write down

$$\sqrt{\rho_i \rho_e} \frac{d\mathbf{v}}{dt} = \operatorname{grad}(p/2) - ne \operatorname{grad} \varphi - \sqrt{\rho_i \rho_e} \operatorname{grad} G - \frac{ne}{\gamma} \mathbf{j} + ne\, [\mathbf{v}\mathbf{B}]. \tag{25}$$

We add two restrictions. As the magnetic field has no source for any plane, the following equation is true:

$$\int_S \mathbf{B}\, d\mathbf{S} = 0. \tag{26}$$

Another restriction is a consequence of the law of impulsive moment conservation. The magnetic field generated inside some volume and acting exclusively upon the substance inside this volume cannot rotate this volume as a whole (Matveyev, 1980).

Last summand in (25) characterizes the density of the magnetic force acting on the conditional charged gas: $\mathbf{f}_M = ne\,[\mathbf{vB}]$. According to the law of conservation of the impulsive moment, the integrated moment of magnetic forces at the volume of the process V equals zero:

$$\mathbf{M} = \int\limits_V [\mathbf{rf}_M]\; dV = 0. \tag{27}$$

Now we carry out the curl operation from both parts of (25):

$$\sqrt{\rho_i\rho_e}\frac{d\Omega}{dt} = -\frac{ne}{\gamma}\,\mathrm{curl}\mathbf{j} + ne\,\mathrm{curl}\,[\mathbf{vB}]. \tag{28}$$

Substituting $\rho_i = m_i n$, $\rho_i = m_i n$, we write out (28) in the following kind

$$rot\mathbf{j} = -a\gamma\frac{d\Omega}{dt} + \gamma\,\mathrm{curl}\,[\mathbf{vB}], \tag{29}$$

where $a = \dfrac{\sqrt{m_i m_e}}{e} = 2.436 \cdot 10^{-10}$ kg/C.

Using Maxwell's equation for magnetostatics: $\mathrm{curl}\,\mathbf{B} = \mu_0\mathbf{j}$, $\mu_0 = 4\pi \cdot 10^{-7}$ H/m is the magnetic constant, we receive:

$$\mathrm{curl}\,\mathrm{curl}\,\mathbf{B} = -a\mu_0\gamma\frac{d\Omega}{dt} + \mu_0\gamma\,\mathrm{curl}\,[\mathbf{vB}]. \tag{30}$$

Using condition $\mathrm{grad}\,\mathrm{div}\,\mathbf{B} = \mathrm{curl}\,\mathrm{curl}\,\mathbf{B} + \Delta\mathbf{B}$ and taking into account that $\mathrm{div}\,\mathbf{B} = 0$, we get

$$\Delta\mathbf{B} = a\mu_0\gamma\frac{d\Omega}{dt} - \mu_0\gamma\,\mathrm{curl}\;[\mathbf{vB}]. \tag{31}$$

Thus, (18, 19) are transformed in the equations (23, 31). The difference of viscous and inertial properties of gases of protons and electrons is the base for the transformation of (31). From (31) it is directly follows that magnetic field emerges from the vorticity changes.

The turbulent viscosity of the proton and electron gases can exceed molecular one by several orders. However, according to the Prandtl's theory of the mixture way (Loytsyansky, 2003) the ratio of turbulent viscosities is same as molecular ones. Therefore final dependence (31) characterizes connection of the plasma vorticity and the magnetic induction not only at laminar movement but also at turbulent movement.

3 Plane model of stationary vortical process

We consider the plane axisymmetric stationary process in the form of vortex-inflow converging to the center (Fig, 1). For this vortex-inflow the radial component of the speed $\mathbf{v}$ is negative ($v_r < 0$) and the rate of inflow $Q = 2\pi r v_r = const < 0$ does not depend on radius (Loytsyansky, 2003). For definiteness we believe that the plasma rotates counter-clockwise and the tangential component of the speed is positive:

For the plane axisymmetric process equation (23) is actual for z-components only since components of $\mathbf{B}$ and Ω at the plane (x, y) identically equal to zero. Further, the radial component of a stationary electric current equals zero $j_r = 0$ as this current does not have sources. Hence,

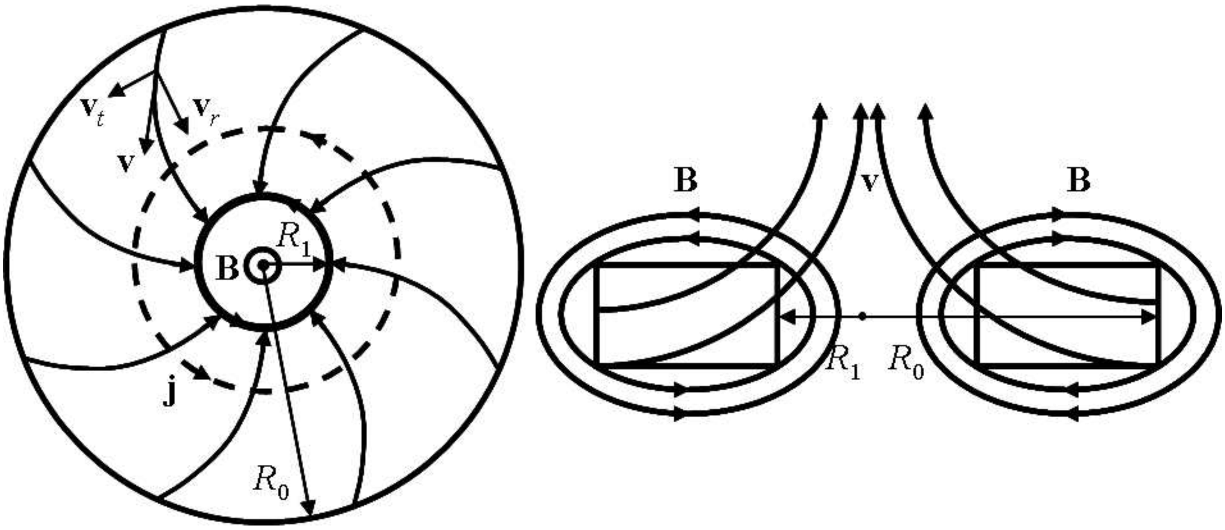

Figure 1: Magnetic field at vortex processes in plasma. On the left is plane model of vortex-inflow, on the right is vertical cross section.

the vector $[\mathbf{j}\,\mathbf{B}]$ is directed along the radius (Fig, 1) and axisymmetric, and for this vector the field curl$[\mathbf{j}\,\mathbf{B}] = 0$. In view of above said, (23) accepts the known form of the equation for movement of a viscous liquid (Loytsyansky, 2003):

$$\frac{d\Omega}{dt} = \lambda\,\Delta\Omega, \tag{32}$$

where hereinafter the index "z" at the corresponding component of vorticity is omitted, $\lambda = \eta/\rho$ is kinematic viscosity of plasma.

We open out the full derivative in (32):

$$\frac{\partial\Omega}{\partial t} + (\mathbf{v}\nabla)\,\Omega = \lambda\,\Delta\Omega. \tag{33}$$

Taking into account the axial symmetry of the convective derivative of the vorticity $\left((\mathbf{v}\nabla)\,\Omega = v_r\dfrac{\partial\Omega}{\partial r}\right)$ and the stationarity of the process $(\partial\Omega/\partial t = 0)$, we write down:

$$\frac{\partial\Omega}{\partial r}v_r = \lambda\,\Delta\Omega. \tag{34}$$

Thus, the vorticity distribution over the area of the plane stationary vortical process in the absence of external magnetic fields depends only on hydrodynamic parameters of the plasma.

We solve (34) in the cylindrical coordinate system:

$$v_r\frac{\partial\Omega}{\partial r} = \lambda\frac{1}{r}\frac{\partial\left(r\frac{\partial\Omega}{\partial r}\right)}{\partial r}. \tag{35}$$

As $Q = 2\pi r v_r$, (35) acquires the following form:

$$k\frac{\partial\Omega}{\partial r} = \frac{\partial\left(r\frac{\partial\Omega}{\partial r}\right)}{\partial r}, \tag{36}$$

where $k = Q/(2\pi\lambda) < 0$ characterizes the ratio of the inflow rate and the plasma kinematic viscosity. After taking the integral by r we reduce the order of differential equation:

$$k\Omega = r\frac{\partial\Omega}{\partial r}.$$

The solution of the last differential equation is the following:

$$\Omega = \Omega_0\left(\frac{r}{R_0}\right)^k. \tag{37}$$

In particular, at the internal border of the process area:

$$\Omega_1 = \Omega_0\left(\frac{R_1}{R_0}\right)^k. \tag{38}$$

Thus, the vorticity increases at the plasma movement to the center. It is obvious that the vorticity Ω_0 at the external border at $r = R_0$ of the process area is same as in the nearest star volume.

4 Magnetic field of the stationary plane vortical process

As the process is plane and axisymmetric, the arising currents are also axisymmetric and have the form of circles (Fig, 1). The corresponding magnetic field has the toroidal form.

The total stream of magnetic induction crossing the plane x, y equals zero according to (26). We consider this stream in the cylindrical system of coordinates:

$$\Phi = 2\pi\int\limits_S Brdr = 0, \tag{39}$$

where it is taken into account that $B_x = B_y = 0$ because of the axial symmetry and also the symmetry relative to the plane (x, y); hereinafter in writing B_z the index "z" is omitted.

Further, we open integral (27) in the cylindrical system of coordinates:

$$\mathbf{M} = \int\limits_S [\mathbf{rf}_M]\, dS = ne\int\limits_{R_0}^{R_1} [\mathbf{r}\,[\mathbf{vB}]]\; 2\pi r\, dr = 0, \tag{40}$$

where the assumption is used that the plasma speed $\mathbf{v} = 0$ outside the process area.

Transforming the double vector product, we have:

$$\mathbf{M} = 2\pi ne\int\limits_{R_0}^{R_1} (\mathbf{v}(\mathbf{rB}) - \mathbf{B}(\mathbf{rv}))\, r\, dr = -2\pi ne\int\limits_{R_0}^{R_1} \mathbf{B}(\mathbf{rv})rdr = 0, \tag{41}$$

where it is taken into account that components $\mathbf{T}$ identically equal zero at the plane (x, y). For the same reason equation (41) is actual only for z-component:

$$2\pi \int_{R_0}^{R_1} B(\mathbf{rv}))\, r\, dr = 2\pi \int_{R_0}^{R_1} B v_r\, r^2\, dr = Q \int_{R_0}^{R_1} B r dr = 0. \tag{42}$$

Thus,

$$\int_{R_0}^{R_1} B r dr = 0. \tag{43}$$

This equality has a clear sense: the total stream of magnetic induction crossing the area of the vortical process equals zero. In combination with (39) this condition means that the total stream of magnetic induction crossing the plane outside the vortical process also equals zero.

Now we will engage (31). First of all, we will transform by known rules the multiplier included in it:

$$\operatorname{curl}[\mathbf{v}\,\mathbf{B}] = (\mathbf{B}\nabla)\mathbf{v} - (\mathbf{v}\nabla)\,\mathbf{B} + \mathbf{v}\operatorname{div}\mathbf{B} - \mathbf{B}\operatorname{div}\mathbf{v}. \tag{44}$$

Here two last summands equal zero because $\operatorname{div}\mathbf{B} = 0$ under Maxwell's equation and $\operatorname{div}\mathbf{v} = 0$ (21). The first summand is also zero: $(\mathbf{B}\nabla)\mathbf{v} = 0$ as the z-component of the speed is not a function of z. Thus,

$$\operatorname{curl}[\mathbf{vB}] = -(\mathbf{v}\nabla)\,\mathbf{B}.$$

This equality is actual only for the z-component. Taking into consideration that magnetic induction depends only on radius $B(r)$, we write down in the cylindrical coordinate system:

$$\operatorname{curl}_z[\mathbf{vB}] = -\frac{\partial B}{\partial r} v_r. \tag{45}$$

Taking into account the stationarity and the axial symmetry of the process, we write down for the vorticity:

$$\frac{d\Omega}{dt} = \frac{\partial \Omega}{\partial t} + \frac{\partial \Omega}{\partial r} v_r = \frac{\partial \Omega}{\partial r} v_r. \tag{46}$$

Equation (23) is actual only for the z-component. Then we make corresponding substitutions (45, 46) and write down:

$$\Delta B = a\mu_0\gamma \frac{\partial \Omega}{\partial r} v_r + \mu_0\gamma \frac{\partial B}{\partial r} v_r. \tag{47}$$

We represent laplacian of the z-component of magnetic induction in the cylindrical system of coordinates and derive the following equation:

$$\frac{1}{r}\frac{\partial\left(r\dfrac{\partial B}{\partial r}\right)}{\partial r} = a\mu_0\gamma\frac{\partial\Omega}{\partial r}v_r + \mu_0\gamma\frac{\partial B}{\partial r}v_r. \qquad (48)$$

We multiply both sides of this equation by r taking into account the equality $Q = 2\pi r v_r$, temporarily introduce parameter $l = \mu_0\gamma\, Q/(2\pi)$, take integrals by r on both sides of this equality and obtaine the following differential equation

$$r\frac{\partial B}{\partial r} = al\Omega + lB. \qquad (49)$$

We again multiply both sides of this equation by r and take integrals by r at the borders of the process area

$$\int\limits_{R_0}^{R_1} r^2\frac{\partial B}{\partial r}dr = al\int\limits_{R_0}^{R_1}\Omega r\,dr + l\int\limits_{R_0}^{R_1} Brdr. \qquad (50)$$

According to (43), the second integral on the right side of (50) equals zero. The integral on the left side is represented as follows:

$$\int\limits_{R_0}^{R_1} r^2\frac{\partial B}{\partial r}dr = \int\limits_{R_0}^{R_1} r^2\,dB = (r^2\,B)\big|_{R_0}^{R_1} - 2\int\limits_{R_0}^{R_1} B\,r\,dr = B_1R_1^2 - B_0R_0^2, \qquad (51)$$

where B_1, B_0 is the magnetic induction at corresponding borders of the process area; (43) is used again. The direction of the magnetic field is determined by the rule of the right screw (Fig, 1).

Not considering in detail, we believe that for the axisymmetric process the following dependence for magnetic induction at the process area boundaries is true

$$B_1R_1^2 = -B_0R_0^2.$$

Accordingly,

$$B_1R_1^2 = \frac{1}{2}\int\limits_{R_0}^{R_1} r^2\frac{\partial B}{\partial r}dr. \qquad (52)$$

We take the remaining integral in (50), using (37)

$$al\int\limits_{R_0}^{R_1}\Omega r\,dr = \frac{al\Omega_0}{R_0^k}\int\limits_{R_0}^{R_1} r^{k+1}\,dr = \frac{al\Omega_0}{(k+2)R_0^k}\left(R_1^{k+2} - R_0^{k+2}\right). \qquad (53)$$

We believe that the vortex-inflow rate is characterized by the factor k: $k < -2$. Then $R_1^{k+2} > R_0^{k+2}$ and the second summand (53) is neglected

$$al \int_{R_0}^{R_1} \Omega r\, dr = \frac{a\,\mu_0\,\gamma\,Q\,\Omega_0\,R_0^2}{2\pi(k+2)} \left(\frac{R_1}{R_0}\right)^{k+2}, \tag{54}$$

where the inverse substitution of l is made. Collecting together the derived formulas, we get

$$B_1 = \frac{a\,\mu_0\,\gamma\,Q\,\Omega_0}{4\pi(k+2)} \left(\frac{R_1}{R_0}\right)^{k} = \frac{a\,\mu_0\,\gamma\,Q\,\Omega_1}{4\pi(k+2)}. \tag{55}$$

Formula (55) has a clear physical sense. The inflow rate should be high enough so that $k < -2$. We remind that the inflow rate is negative, as the inflow (Loytsyansky, 2003) is considered as the negative outflow. Magnetic induction in the central area of the vortical process is proportional to the plasma conductivity, to the rate of inflow and to the plasma vorticity at the internal border of the vortical process region.

5 Magnetic induction of arising magnetic field

It is obvious that it is possible to speak about reality of the model only if the suggested mechanism gives magnetic fields corresponding to the observable ones. We make comparison for sunspots which are the closest prototype of the suggested model of the process. Here we will only calculate dependence of a magnetic induction from the plasma temperature.

We use (55) for estimations. For the axisymmetric currents the z-component of vorticity Ω equals (Loytsyansky, 2003):

$$\Omega = \frac{\partial v_t}{\partial r} + \frac{v_t}{r},$$

where v_t is the tangential component of the plasma speed.

At the solid-state rotation the tangential speed is proportional to the distance from center: $\partial v_t/\partial r = v_t/r$; the vorticity is constant everywhere: $\Omega\,(r) = 2v_t/r = const.$ At vortex-free rotation the vorticity equals zero $\partial v_t/\partial r = -v_t/r$, $\Omega\,(r) = 0$ and the plasma tangential speed is inversely proportional to the distance. These two extreme cases correspond to either very high viscosity ($v_r = 0$) or to the full absence of viscosity. For estimations we will take the average case $\partial v_t/\partial r = 0$. Then $\Omega = v_t/r$.

Using also relation $Q = 2\pi r\, v_r(r)$ and the arbitrary size $k = -2.5$ we get:

$$B_1 = a\,\mu_0\,\gamma\,v_t(R_1)\,|v_r(R_1)|\,. \tag{56}$$

The unit is tesla:

$$|B| = |a|\cdot|\mu_0|\cdot|\gamma|\cdot|v|^2 = \frac{\text{kg}}{\text{C}}\frac{\text{H}}{\text{m}}\frac{\text{mho}}{\text{m}}\frac{\text{m}^2}{\text{s}^2} = \text{tesla} = 10^4\ \text{G}.$$

For estimations we believe that modules of tangential and radial speeds of the stellar plasma at the internal border of the inflow area equal each other: $|v_r(R_1)| = |v_t|$. Then $v_t^2 = v^2/2$ and

$$B_{R_1} = a\,\mu_0\,\gamma\,v^2/2. \tag{57}$$

We consider temperature T as the main parameter and believe that Coulomb's multiplier in (17) equals $L_e = 3.5$:

$$\gamma = 4.37 \cdot 10^{-3} \cdot T^{3/2}.$$

To estimate the maximal plasma motion velocity we use the Bernoulli theorem:

$$p + \rho v^2/2 = p_0,$$

where p and p_0 are magnitudes of pressure inside and outside the vortical process, respectively. The ratio of p_0 to p we can find from the equation of adiabat:

$$\left(\frac{p}{p_0}\right)^{n-1} \left(\frac{T}{T_0}\right)^{n} = const,$$

where n is the adiabatic index which is equal to 5/3 for the completely ionized plasma. The temperature of a sunspot at the photosphere surface is approximately $b = 1.5$ times lower than the temperature of a quiet Sun at the photosphere surface. We consider that the ratio of temperatures inside and outside the sunspot at the depth of several thousand kilometers underneath the photosphere is considerably less and equals $b \equiv 1 + \alpha = 1.1$ ($\alpha = 0.1$). Hence, the decrease of temperature by a factor of 1.1 in accordance with equation of adiabat results in the decrease of pressure $b^{5/2} \approx$ 1.25 times, i.e.

$$p = b^{-5/2} p_0 \approx 0.75 p_0, \quad v^2 = 2(p - p_0)/\rho = 0.5 p_0/\rho. \tag{58}$$

Taking into consideration the Mendeleyev–Clapeyron equation, the expression for the plasma velocity squared is represented as follows:

$$v^2 = 5\alpha \frac{p_0}{\rho_0} = 5\alpha \frac{RT}{\mu} = \frac{5\alpha \cdot 8.31 \cdot T}{0.5 \cdot 10^{-3}} = 8.31 \cdot 10^4 \alpha T \, \frac{\mathrm{m}^2}{\mathrm{s}^2}, \tag{59}$$

where we have taken into account that molar mass μ of the ionized hydrogen plasma is half as large as molar mass of monatomic hydrogen.

Collecting the obtained results:

$$B_{R_1} = 2.44 \cdot 10^{-10} \, \frac{\mathrm{kg}}{\mathrm{C}} \cdot 4 \cdot 3.14 \cdot 10^{-7} \, \frac{\mathrm{H}}{\mathrm{m}} \cdot 8.31 \cdot 10^4 \alpha T \, \frac{\mathrm{m}}{\mathrm{s}} \cdot 4.37 \cdot 10^{-3} \, \frac{\mathrm{mho}}{\mathrm{m}},$$

$$B_{R_1} = 5.55 \cdot 10^{-14} \alpha T^{5/2} \, \mathrm{tesla} = 5.55 \cdot 10^{-10} \alpha T^{5/2} \, \mathrm{G}. \tag{60}$$

In the range of temperatures 50 000 – 200 000 kelvin magnetic induction varies in the range B_{R_1} = 31 – 990 G. Magnetic fields of such intensity are frequently observed in stars, in particular, they are characteristic for sunspots.

6 Conclusion

The magnetic field arises in vortical processes in the stellar plasma even if initially it was absent. For the simplified plane model it is shown that the maximal magnetic induction 31 – 990 G corresponds to the range of plasma temperatures 50 000 – 200 000 kelvin. Such induction is typical of sunspots.

Application of the suggested mechanism in construction of the theory of sunspots is very promising. For this purpose it is necessary:

- to systematize the observations data and to generalize the existing representations by the spatial structure of sunspots and their magnetic fields;

- to construct a three-dimensional model of a sunspot on the common basis of the suggested mechanism and "dynamo" mechanisms;

- to compare the natural and modeled data.

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