Radiation Mediated Shocks

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Existence of Radiation Mediated Shocks

- Fast shocks' downstream pressure:
  $2n_d T_d$ (particles) ; $a_{BB} T_d^4/3$ (radiation)
- For fast enough shocks,
  $\beta \gg 3 \times 10^{-4} \left(\frac{n}{10^{20}\ \mathrm{cm}^{-3}} \right)^{1/6}$,
  the DS pressure is dominated by radiation, if
  $\tau \gg 1$ (i.e. optically thick system) $\Rightarrow$ RMS.
- The DS equilibrium temperature:

$$T_d = \left(\frac{315}{4\pi^2} n_u \varepsilon \hbar^3 c^3 \right)^{1/4} \approx 0.16 \left(\frac{\varepsilon}{10\mathrm{MeV}} \frac{n_u}{10^{15}} \right)^{1/4} \mathrm{keV}$$

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Shock Breakout

- Shocks running in CC SN are expected to be RMS.
- Close to the photosphere $T_d \lesssim 200\text{eV} \Rightarrow$ expected SN precursor in thermal UV/X-ray.[1]
- XRF060218/SN2006aj[2] and SN 2008d[3] found early after the explosion, non thermal > 10keV spectrum.
- Is the difference related to the structure of the shock?[4]

Other explanations: photon acceleration [Wang et al. 07, 08], not a breakout [Li 08, Mazzali et al. 09]

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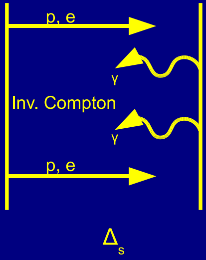
RMS velocity transition

Cold Upstream

$\Rightarrow$

$v, \ \Gamma, \ \varepsilon$



Radiation dominated Downstream

$\Rightarrow$

$$U_{rad} \gg U_{pl}$$

$$U_{rad} \sim \Gamma n_p m_p c^2$$

or

$$U_{rad} \sim \frac{1}{2}\beta_u^2 m_p c^2$$

$$t_\gamma \approx \frac{\Delta_s}{\lambda_\gamma}\frac{\Delta_s}{c} \approx t_e \approx \frac{\Delta_s}{v} \Rightarrow \frac{\Delta_s}{\lambda_\gamma} \approx \frac{c}{v}$$

$$L_{\rm dec} = (\sigma_T n \beta)^{-1}$$

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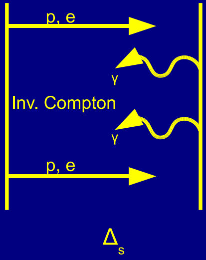

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Physical Assumptions

- Steady state shock
- p, e^-, e^+ treated as one fluid (plasma)

$$\frac{t_{pl}}{t_{scat}} \approx 10^{-11} \frac{n_\gamma}{n_e} n_{15}^{1/2}$$

- Radiation mechnisms:
  - Compton scattering
  - Bremsstrahlung
  - Pair production and annihilation

Scaling relations:

Only Bremsstrahlung self absorption depends on n_u!

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NR RMS

- Steady state, 1D, self consistent solutions of: Radiation transport and conservation of P, F and n_p.
- Numerical solutions, analytic estimates
- NR: Numerical solution by Weaver (1976): diffusion approximation, Wien spectrum.
- Shock structure:
  - Deceleration on a scale of $\beta^{-1}\lambda_c$
  - Production of downstream equilibrium radiation:
    - High density, low velocity: all in equilibrium
    - Low density, high velocity:
      T increases inside the shock velocity transition,
      Slow thermalization follows until $T = T_d$

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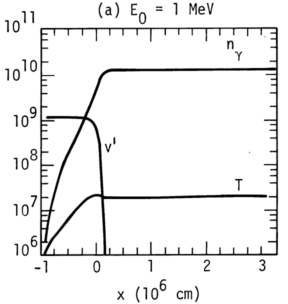
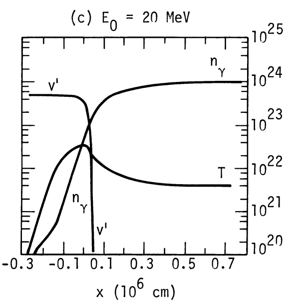
Shock structure [5]





Low n_u, high β_u High n_u, low β_u

[5] Weaver 1976

Analytic estimates: thermalization width

n_γ: Production/Diffusion (Wein equilibrium)
Thermalization length:

$$L_T \sim \beta c \frac{n_{\gamma,\mathrm{eq}}}{Q_{\gamma,\mathrm{eff.}}};\ Q_{\gamma,\mathrm{eff}} = \alpha_e n_p n_e \sigma_T c \sqrt{\frac{m_e c^2}{T}} \Lambda_{\mathrm{eff}} g_{\mathrm{eff}}$$

High temperatures inside the shock transition:

$$\beta_u > 0.07 n_{15}^{1/30} \left(\Lambda_{eff} g_{eff}\right)^{4/15} \Rightarrow L_T > \Delta_s \Rightarrow T_s > T_d$$

$$\Lambda_{eff} \approx \log \frac{T}{\nu_a(@N_{coll} = m_e c^2/4T)}$$

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Analytic estimates: T_s [6]

$$n_{\gamma,s} \approx Q_\gamma(T_s, n_d)\frac{1}{3 n_d \sigma_T \beta_d c} \text{ (Diffusion)}$$

$$n_{\gamma,s} T_s \approx \frac{12}{7}\varepsilon n_u \text{ (Momentum cons.)}$$

Velocity - temperature relation:

$$\beta_s \approx 0.2 \left(\frac{\Lambda_{eff}}{10}\frac{g_{eff}}{2}\right)^{1/4} \left(\frac{T_s}{10\mathrm{keV}}\right)^{1/8}$$

$n = 10^{15}$ cm^{-3}; $\varepsilon = 30$ MeV ($\beta = 0.25$)

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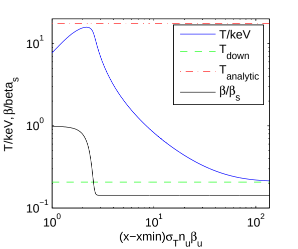

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Application to X-ray Breakout[8]

- Envelope density $\rho \propto \delta^n$, $\delta = (1 - r/R)$ (n=3, 3/2 for radiative (BSG, WR), convective (RSG))
- Shock velocity (interpolating ST-Sakurai)[7]: $\mathrm{v}_s \approx \left(\frac{E}{M}\right)^{1/2} \delta^{-0.2n}$
- $\delta E \sim 4 \times 10^{46} \beta_s^2 R_{12}^2 (\kappa/\kappa_T)^{-1} \tau_{0.5}$ erg

Post-shock thermal energy $U_{rad} = \frac{18}{7}\rho \mathrm{v}_s^2$

$10\left(\frac{E_{51}}{M_\odot}\right)^{1/2} \approx 0.24c$

$\Rightarrow$

Optical depth

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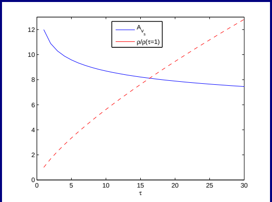

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Existence of RRMS

- SN shock breakout can reach mildly relativistic ($\Gamma\beta \gtrsim 1$) velocities: GRB980425/SN1998bw[9], GRB030329/SN2003dh[10], GRB031203/SN2003lw[11], XRF060218/SN2006aj[12].
- A relativistic jet penetrating through a stellar mantle[13]: $\Gamma_s \approx \Gamma_j/2$ (decelerating shock) $\Gamma_h \approx 2$ (forward shock into the mantle)

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Solving RRMS structure[14]

- Anisotropy $\Rightarrow$ ~~Diffusion Approximation~~ $\rightarrow$ full transport.
- Relativistic corrections to the Scattering (KN) $\Rightarrow$ transport is frequency dependent.
- e^+e^- pairs.
- Relativistic corrections to production mechanisms.

Assumptions:

p, e^- and e^+ are coupled and equilibrated.

Complications of the solution

- Full transport with hydrodynamics, no definite boundary conditions
- Sonic points crossings
- Solution by iterations of radiation/hydro

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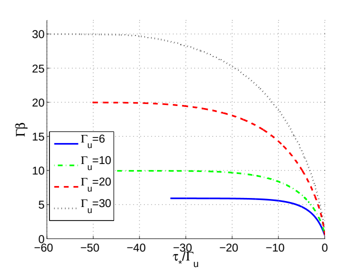

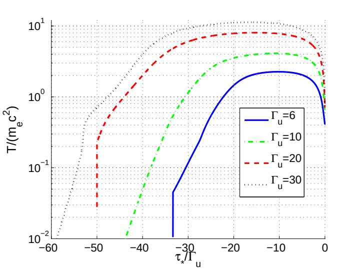

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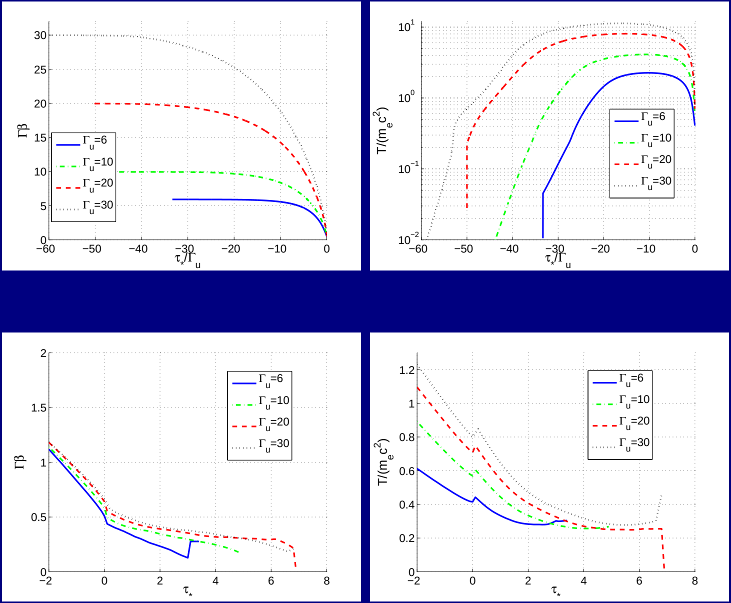
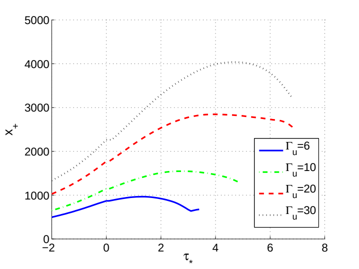
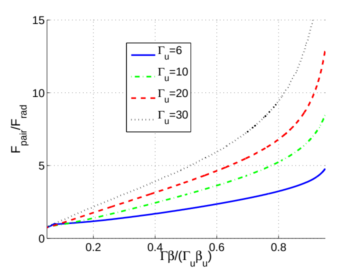


Pair domination and subsonic regime

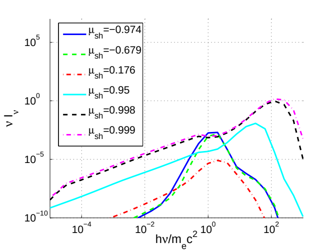


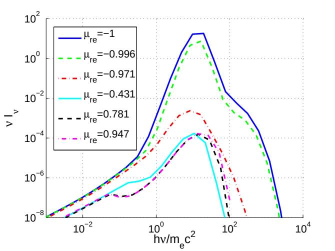


- $n_+ \gg n_p$; $T \sim m_e c^2 \Rightarrow$ relativistic speed of sound $c_{ss} \sim c/\sqrt{3} > \mathrm{v}_d$

Radiation spectrum within the shock

$\Gamma_u = 10$:

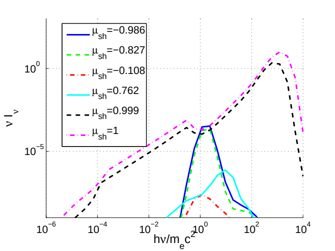


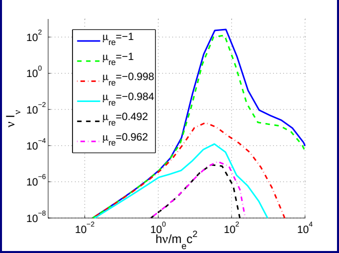


Shock frame

Rest frame of the plasma

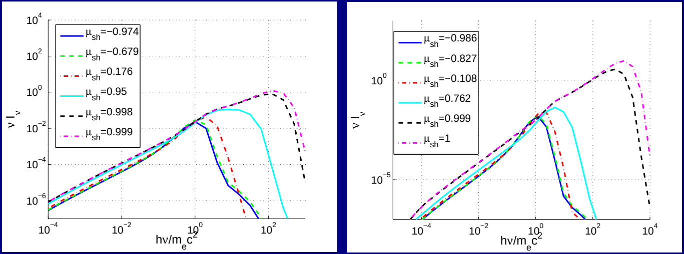
$\Gamma_u = 30$:



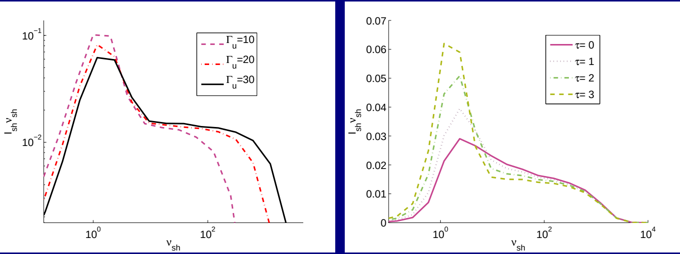


Radiation spectrum in the DS

$\Gamma_u = 10$ $\Gamma_u = 30$



Integrated spectrum:



Analytic model for the Immediate DS

The immediate DS supplies the photons stopping the plasma:

Diffusion/Production of γ's ($\beta \sim 1/3$):

$$\frac{n_\gamma}{n_l} \approx 2.5 \left(\frac{\Lambda_{\text{eff}}}{15}\right)^2 (3\beta_d)^{-2}$$

Compton-Pair equilibrium:

$$\frac{n_\gamma}{n_l} \approx 0.5 \frac{m_e c^2}{T}$$

$\Rightarrow T_s \approx 200\text{keV}$ (average over $\sim$ 3 optical depths!)

Num. results compared to CPE

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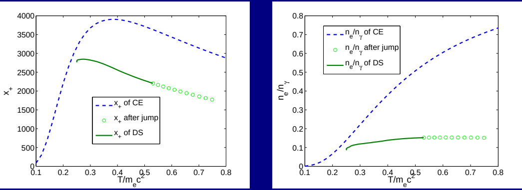

Analytic model for the transition

- The structure is set by photons from the immediate DS ($h\nu \sim m_e c^2$ in the shock frame) penetrating deep into the US.
- In the transition $T \sim \Gamma m_e c^2$ (Compton "equilibrated")
- KN correction for Compton: $\sigma/\sigma_T \sim \Gamma^{-2}$
  Corrections for $\sigma_{\gamma\gamma}$ behave the same

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Analytic model for the non thermal beam

The beam originates from Compton scattering of US going $h\nu \approx m_e c^2$ photons on the deceleration profile. In the shock frame:
$h\nu' \approx \Gamma^2 m_e c^2$ (since $T \sim \Gamma m_e c^2$)

A simplifies expression:

$$\hat{\nu}_{sh} I_B(\hat{\nu}_{sh}, \theta_{sh}) \propto \hat{\nu}_{sh}^{1-\alpha_1} \Theta(\theta_{sh}^{-1} - \hat{\nu}_{sh}^{1/2}) \Theta(\hat{\nu}_{max} - \hat{\nu}_{sh})$$

$\alpha_1 \approx 1$, $\hat{\nu} \equiv h\nu / m_e c^2$

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Summary

- NR RMS
  - High T within the shock transition $\beta_s \approx 0.2 \left(\frac{T_s}{10\mathrm{keV}}\right)^{1/8}$
  - SN breakout: $\beta_s \gtrsim 0.1 \Rightarrow$ X-ray emission $\gtrsim 10\mathrm{keV}$
    Expected high energy photon component from early breakouts
- RRMS
  - We derived the numerical steady state solution + analytical approximations
  - Immediate DS: $T \sim m_e c^2$
  - Subsonic regime, weak subshock
  - High energy power law beam,
    $\nu I_\nu \propto \nu^0$, $h\nu_{max} \approx \Gamma_u^2 m_e c^2$,
    beamed towards the DS $\theta_B \sim \Gamma_u^{-1}$

Summary

- NR RMS
  - High T within the shock transition $\beta_s \approx 0.2\left(\frac{T_s}{10\mathrm{keV}}\right)^{1/8}$
  - SN breakout: $\beta_s \gtrsim 0.1 \Rightarrow$ X-ray emission $\gtrsim 10\mathrm{keV}$
    Expected high energy photon component from early breakouts
- RRMS
  - We derived the numerical steady state solution + analytical approximations
  - Immediate DS: $T \sim m_e c^2$
  - Subsonic regime, weak subshock
  - High energy power law beam,
    $\nu I_\nu \propto \nu^0$, $h\nu_{max} \approx \Gamma_u^2 m_e c^2$,
    beamed towards the DS $\theta_B \sim \Gamma_u^{-1}$