# Statistical study of spin dynamics in precessing binary black holes in eccentric orbits

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**Abstract** We study the evolution of spins of binary black holes taking eccentricity into account. We are interested in determining the probability that these binaries may get locked in spin-orbit resonance configurations. We numerically evolve a large set of binaries starting from a large separation by using post-Newtonian approximation. The initial spin distribution is taken to be isotropic as is expected in many astrophysical environments. We find that a large fraction of the binaries get locked into or oscillate about the spin-orbit resonance configuration towards the end of inspiral. This can be tested in future observations which, using our results, will allow us to deduce the initial spin distributions.

**Keywords:** Binary Black Holes, Spin-Orbit Resonances, Gravitational Waves, Spin Morphology, Eccentric Binaries

## 1. Introduction

The black hole (BH) spin is an important parameter in binary black holes (BBHs). It has a significant effect on the emitted gravitational waves. The spin induced effects are more dramatic when the two BH spins in a BBH are not exactly aligned or anti-aligned with the orbital angular momentum. Taking the spins into account introduces six additional parameters in the binary system. Resultantly the extraction of parameters from the observed gravitational waves becomes substantially more complicated [1].

There are three phases in the evolution of BBHs: inspiral, merger and ringdown. The inspiral phase can be handled by using the post-Newtonian (PN) approximation [2-4]. The spins and the orbital angular momentum vectors significantly evolve during this phase although the spin magnitudes remain unchanged. Due to the general relativistic spin-orbit and spin-spin coupling, the spin vectors as well as the orbital angular momentum undergo precession about the total angular momentum $\vec{J}$ which remains fixed on small time scales and changes only on the radiation reaction time scale. The important time scales in the PN limit are:

(i) Orbital time scale $t_{orb}$ ,
(ii) Precession time scale $t_P$,

(iii) Radiation-reaction time scale $t_{GW}$.

These obey, $t_{orb} \ll t_P \ll t_{GW}$. The directions of spin and orbital angular momentum change on the precession time scale as well as on radiation-reaction time scale.

We are interested in determining the statistical distribution of spins of BBHs near merger assuming an initial distribution at large separation. The binary can get locked in a spin-orbit resonance depending on its initial parameters. In this configuration the spin vectors and the orbital angular momentum vector lie in the same plane and precess about $\vec{J}$ at the same resonant frequency. In this study we shall determine the probability that the binaries may get trapped in a spin-orbit resonance at late inspiral. It has been shown earlier that if the distribution of spins is initially isotropic, i.e. the spins of BHs in BBHs have equal probability to point in any direction, then the distribution remains isotropic during evolution. However it undergoes significant change if it is initially non-isotropic [4]. In general the initial spin distribution of the binary partners is expected to be partially aligned and depends on the astrophysical environment of the binary. Through our study we can deduce the initial spin distribution once we have a sufficiently large sample of gravitational waves from BBHs and hence can deduce the nature of the astrophysical environment of the binary system [5]. These predictions can be further tested by multi-messenger astronomy since the populations of BH-NS (Neutron Star) and NS-NS binaries are related to those of BBHs.

## 2. Evolution of Binary Spins

Let $\vec{S}_1$ and $\vec{S}_2$ denote the spin vectors of the two BHs and $\vec{L}$ be the orbital angular momentum vector. Consider a system in which the $\hat{z}'$ axis points along $\vec{L}$. Let $\theta_1$ and $\theta_2$ be the polar angles of the two spin vectors and $\Delta\varphi$ be their azimuthal angle separation. In a spin-orbit resonance (SOR) configuration, the three angular momenta vectors of BBH occupy a planar configuration with $\Delta\varphi = 0, \pi$ [3,6,7]. Hence we can split the binaries into the following three classes or morphologies:

1. Circulating: The three spin vectors are non-planar, freely precessing and $\Delta\varphi$ can take any value.
2. SOR-0: The spin vectors and orbital angular momentum vector librate about co-planar configuration with $\Delta\varphi = 0$ on precession time scale.
3. SOR-180: The spin vectors and orbital angular momentum vector librate about co-planar configuration with $\Delta\varphi = \pi$ on precession time scale.

We are interested in determining the relative probability of BBHs to be in these three classes at near merger. The initial distribution of spins is taken to be partially aligned. This question has earlier been addressed by assuming that the eccentricity $e$ is zero ([6]-[8]). However this is not true in reality. The eccentricity is expected to become smaller during inspiral, however beyond a certain point it is expected to start increasing [9, 10]. Furthermore even if we set $e = 0$ initially, it is expected to become non-zero during subsequent evolution. Here we examine the general case in which eccentricity is non-zero.

The basic evolution equations for the spin vectors are given in ([6]-[8]). Here we use the radiation reaction equations at 3PN order in order to evolve the binary from $a = 1000M$ to $a = 10M$, where $a$ is the binary separation. The PN approximation breaks down at smaller $a$ values.

# 3. Results

The evolution of eccentricity for a set of BBHs is shown in Fig. 1. Depending on the values of initial eccentricities, the evolution of eccentricity during inspiral exhibits three distinct patterns: (i) monotonic decay of eccentricity, (ii) monotonic growth of eccentricity, and (iii) growth of eccentricity after decaying to a minimum. In Fig. 1, the initial eccentricities correspond to those values for which $de^2/dt$ vanishes at 2PN order. The eccentricities cannot be decreased beyond these values. We evolve 4000 such maximally spinning eccentric binaries with mass ratios q = 0.40, 0.60, 0.80, and 0.95 (1000 eccentric binaries for each mass ratio), where $q = \frac{m_2}{m_1} \leq 1.$ In Fig. 1, the median of eccentricity evolution for each set of 1000 binaries are plotted. Since, the growth of eccentricity is a spin induced phenomenon, there is no significant difference in eccentricity evolution for the four different mass ratio cases. The eccentricities rise to $10^{-3}$ towards the end of inspiral. Such eccentricities are not negligible in numerical relativity simulations.

In Fig. 2, we show the morphology obtained for the evolved spin configurations from Fig.1 at the initial (a = 1000M) and final (a = 10M) orbital separation. At a = 1000M, most of the binaries are in the circulating morphology i.e. freely precessing. The morphologies predict average behaviour of binaries in a precessional time period which preserve information about initial configurations. During inspiral, the morphologies of spin configurations evolve slowly. At the separation a = 10M, a large fraction of binaries from the circulating morphology transit to the resonant morphologies. The behaviour of morphology transition of eccentric binaries is similar to that of circular binaries despite significant rise in eccentricity. This implies that morphology classification is a useful tool to study spin dynamics in eccentric binaries too which eventually would give information about the initial distribution of spins. The morphology evolution of eccentric binaries is similar to that of circular binaries owing to the fact that eccentricity growth is not dependent on the mass ratio, whereas mass ratio is a dominant parameter of the dynamics spinning binaries.

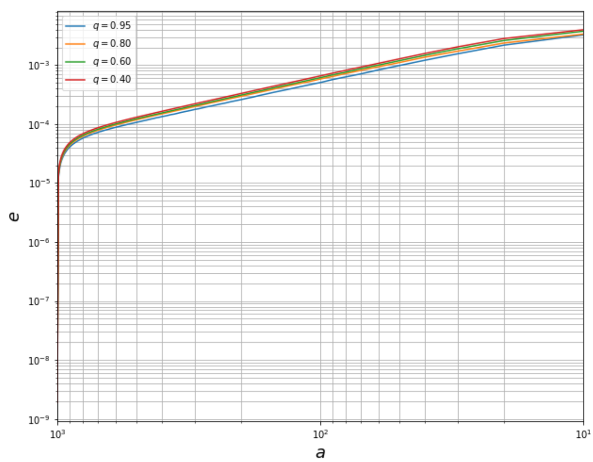


***Fig1.*** *The evolution of eccentricities during inspiral. The binaries with four mass ratios q = (0.4, 0.6, 0.8, 0.95) are maximally spinning. We observe significant growth in eccentricity near merger of binaries.*

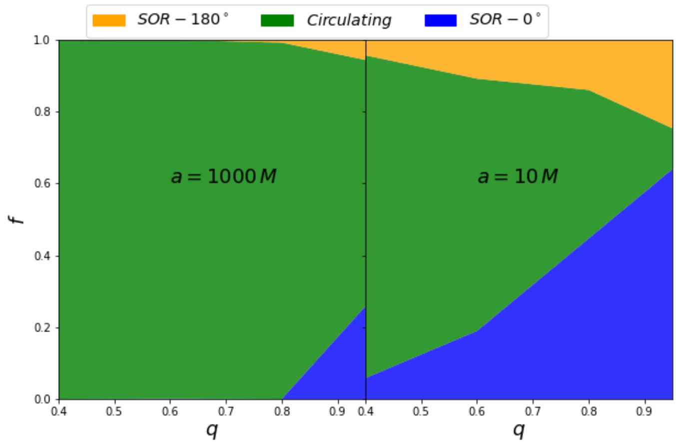


***Fig2.*** *In this figure, we show the evolution of morphologies of binaries during inspiral. As in case of circular binaries, the freely precessing binaries in eccentric orbits also gradually transit to resonant morphologies (SOR-180 and SOR-0) where the angular momentum vectors of binaries occupy a planar configuration or oscillate about the planar configurations.*

## 4. Conclusion

We have shown that a significant fraction of black hole binaries get locked up in the spin-orbit resonance configurations or oscillate about them towards the end of inspiral. This result has earlier been obtained by ignoring the eccentricity of the binary orbits. We find that statistically the result remains unchanged even if we include the effect of eccentricity.

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