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Light curve analysis of rapidly variable astronomical objects in the MANIA experiment

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Abstract. We present methods and algorithms of plotting, modeling and statistical analysis of light curves in the experiment MANIA, and illustrate their using in processing the results of observations of Nova Persei (GRO J0422+32) made with the 6 m telescope in 1992. The algorithms are realized in the language C (Linux) using the graphic package Libsx. Variability of Nova Per was analysed. It was found that the most characteristic duration of flashes is about 0.5 s and that there exists inverse correlation between the amplitude and the duration of flashes.

Key words: stars: X-ray: stars: statistics —stars: individual: Nova Persei

1. Introduction

The X-ray nova GRO J0422+32 (Nova Persei) was discovered near the maximum of the flash in August, 1992. Since that time quite a number of investigations of the object have been made including those carried out in the frames of the experiment MANIA (Shvartsman, 1977; Shvartsman et al., 1997). In the 6 m telescope observations a hardware–software complex of the experiment which incorporated a 2-channel photometer placed at the N1 focus of the telescope, a recording system "Quantochron" (Zhuravkov, Plokhotnichenko, 1991, Zhuravkov et al., 1992) and special software (Plokhotnichenko, 1983) was used. A rapid analysis of the results revealed a variability of brightness in a wide time range (Beskin et al., 1994, 1995). In the present paper we describe methods of construction and analysis of light curves in the MANIA experiment and their application to study of variability of GRO J0422+32.

2. Observational data

In the observations of Nova Persei high precision was achieved in determining the moments of recording separate photons and the photometric band (B, V) in which they were detected. The high information capacity of the observational data permits different methods of analysis to be used in the processing: search for stochastic variability, in the time range 10^{-6} s to 100 s, search for and investigation of periodic variability of radiation, as well as comprehensive study of light curves.

3. Plotting of light curves

Study of light curves is the most adequate method of investigation of the fine time structure of radiation of variable objects with a moderate time resolution (1 ms – 100 ms). The original data for the procedure of plotting light curves are the moments of detected photon arrival in two colour bands. A photocount at the photometer output corresponds to each detected photon. A sample of moments are superimposed on a homogeneous scale of time windows and the amount of photocounts N_i falling within the windows is determined.

The minimum size of the window is defined by the intensity of the quantum flux. To achieve an accuracy of 20% in the measurement, it is necessary to have not less than about 25 quanta in the window. The flux of photocounts from the star Nova Persei is ~ 3000 phcts/s in the channel V and ~ 2000 phcts/s in B, which enables the light curves to be plotted with a resolution of 10 ms.

In the process of observations, when the information is stored on magnetic disks the acquisition of new data is stopped. When searching for stochastic variability at short times (<1 ms), these discontinuities can be neglected. However, the analysis of the light curves at longer times requires the gaps to be reconstructed.

One of the aims of the programme was filling the gaps by modelled fluxes of photocounts whose mean intensities correspond to interpolated values determined from the regions of the real light curve which are adjacent to the gaps. Proceeding from the as-

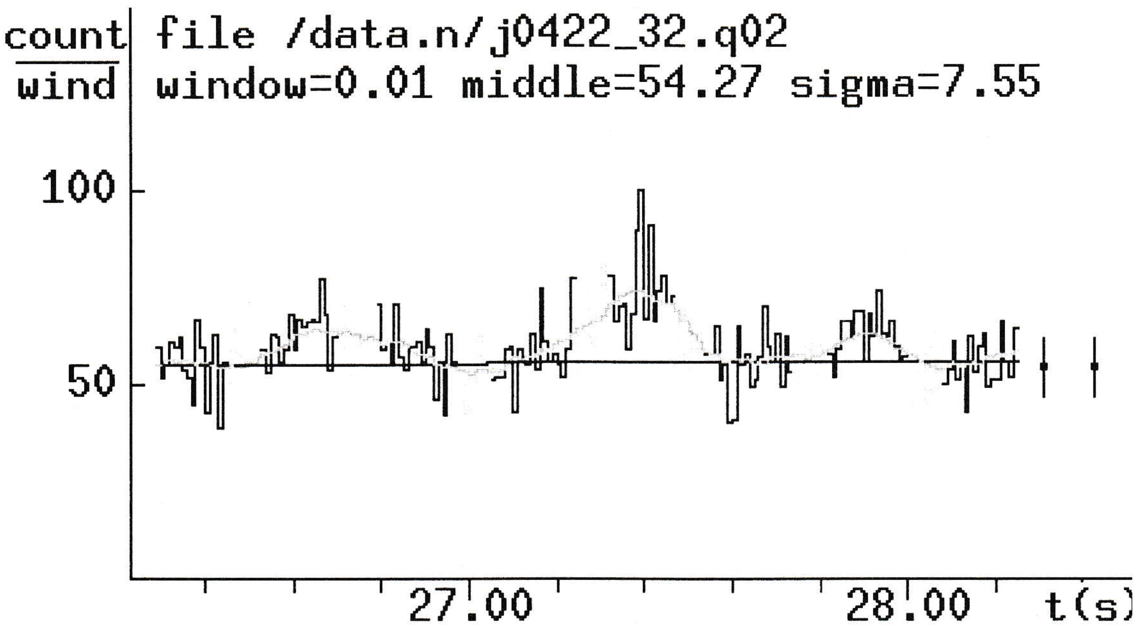

Figure 1: *A light curve portion with filled intervals.*

sumption that the series of time moments of quantum recording are the realization of the Poissonian process with variable intensity, the model series was constructed with the aid of the generator of random numbers, as the Poissonian one. This assumption was repeatedly tested in the course of the MANIA experiment, it was ascertained to be true with a high accuracy provided that the light recording equipment works well (Plokhotnichenko, 1993).

4. Modeling of Poissonian fluxes

Let us examine the mathematical aspect of generating a model Poissonian flux (Koks, Lewis, 1969). It is characteristic for this flux that the probability of appearance of an event in a small time interval $(t, t+h)$ is independent of the presense of any previous event, but it depends only on the intensity of the flux of events λ and the length of the interval h:

$$P\{N_{t,t+h} = 1\} = \lambda h + o(h), \quad \text{at} \quad h \to 0.$$

From this it follows that the events are distributed uniformly in the probability interval (0,1). The formula of the distribution of intervals between the events in the Poissonian flux is as follows:

$$P\{\tau > \Delta t\} = e^{-\lambda \Delta t},$$

here τ is the interval between the moment of the beginning of count and the appearance of the first event, Δt is the time interval in which not a single event occurs. Thus the length of the time interval between the current moment (the previous event may be chosen as the current moment) and the next event is determined by the quantity

$$\Delta t = -\frac{\ln P}{\lambda}.$$

We derive the probability P of the next event from the generator of random numbers, while the value of λ is computed as the expected quantum flux intensity by means of interpolating the intensity in the gap from the adjacent regions of the light curve. Next, by computing the accumulated sums of the intervals

$$t_j = t_{j-1} + \Delta t_j,$$

we construct a time series of model events similar to the real one and work with it according to the general scheme.

5. Check of the quality of modeling

To ascertain the fit of the model and real fluxes of quanta, we compare the sample variances of the model flux (and also the real fluxes from the comparison star and the object) with the theoretical variance of the Poissonian process of constant intensity, which is known to be equal to the average number of events (Gerasimovich, Matveeva, 1978).

The sets of data from the comparison star as a source, which radiation is obviously devoid of intrinsic variability, must be Poissonian to a high accuracy. This is why the sample variances of the model flux constructed by the data from the comparison star and of the real flux must be close to the Poissonian variance.

Table 1: *Sample variances derived from the comparison star data (S) and from the object (O) Nova Persei, with a window of 0.01 s*

S/O	N^o	$< N_r >$	D_r	D_m	D_{r+m}	$F(\nu_r, \nu_\infty)$	$F(\nu_m, \nu_\infty)$	$F(\nu_{r+m}, \nu_\infty)$
S	01	49.788	51.093	52.735	52.040	1.026	1.059	1.045
S	02	49.824	52.647	51.764	52.063	1.057	1.039	1.045
S	03	52.855	55.047	55.235	55.184	1.041	1.045	1.044
S	04	52.353	54.714	56.195	55.877	1.045	1.073	1.067
O	01	57.483	75.966	71.855	74.384	1.322	1.250	1.294
O	02	55.260	79.432	76.401	78.053	1.437	1.383	1.412
O	03	54.269	76.367	77.492	77.115	1.407	1.428	1.421
O	04	54.988	76.535	74.762	75.394	1.392	1.360	1.371
O	05	56.354	74.920	74.370	74.677	1.329	1.320	1.325
O	06	56.255	81.076	78.082	80.290	1.441	1.388	1.427
O	07	54.898	86.216	82.398	84.837	1.570	1.501	1.545
O	08	53.810	69.608	68.890	69.682	1.294	1.280	1.295
O	09	53.804	69.378	68.023	68.713	1.289	1.264	1.277
O	10	59.638	88.385	88.409	87.775	1.482	1.482	1.472
O	11	55.572	77.550	73.551	77.062	1.395	1.324	1.387
O	12	52.846	72.176	71.259	72.022	1.366	1.348	1.363

Notes:
N^o is the ordinal number of the sample;
$< N_r >$ is the averaged number of photons in the window;
D_r is the sample variance of the real light curve;
D_m is the sample variance of the model light curve;
D_{r+m} is the mixed variance;
$F(\nu_r, \nu_\infty)$ is the ratio of the sample variance of the real flux to the Poissonian variance;
$F(\nu_m, \nu_\infty)$ is the ratio of the model flux variance to the Poissonian;
$F(\nu_{r+m}, \nu_\infty)$ is the ratio of the mixed variance to the Poissonian.

If the object under study is physically variable at times of milliseconds to tens of seconds, the sample variance must then be greater than the Poissonian one with equal mean intensities, and the filling of the gaps of the object light curve with the Poissonian noise must result in a certain drop of variance.

As the null hypothesis assume $\sigma^2 = \sigma_0^2$. Here σ_0^2 is the theoretical (Poissonian) variance, σ^2 is the sample variance (one of the above indicated). The criterion is based on the statistics $y = (n-1)S^2/\sigma_0^2$, where n is quantity of elements in the sample, S^2 is the sample variance. The critical region of rejecting the hypothesis at the given level of significance α is $y < \chi^2_{\alpha/2}$, $(m = n-1)$. With a large m χ^2 distribution becomes normal. The corresponding quantile of the standardized normal distribution is determined by the formula (Korn, Korn, 1974):

$$U_{\alpha/2} = \frac{\sqrt[3]{\chi^2_{\alpha/2}/m} - 1 + 2/9m}{2/9m}.$$

In our case $m > 5000$. From the quantile $U_{\alpha/2}$ the corresponding probability is found; thus, the hypothesis, that the sample examined belongs to the same general aggregation the Poissonian flux with constant intensity does, is checked.

Table 1 presents the sample variances obtained from real, model and mixed light curves as well as their ratio to the Poissonian variance. Analysis has shown that in the comparison star samples a slight variability is detected reaching 2.6σ in the 2nd sample. This suggests that the hypothesis that the variances coincide can be rejected at a significance level of 0.5%. This variability is probably due to atmospheric instability.

In the samples accumulated from the object a much stronger variability (minimum — 12σ, maximum — 25.3σ) is revealed. This variability cannot be explained by atmospheric instability alone. It can be seen that the filling of the gaps, as was to be expected, reduced the variance of mixed fluxes, which suggests that the flash flux was "thinned" with the model one. From this it can be inferred that, as a whole, the model Poissonian fluxes that fill the gaps and the corresponding real fluxes are statistically indistinguishable.

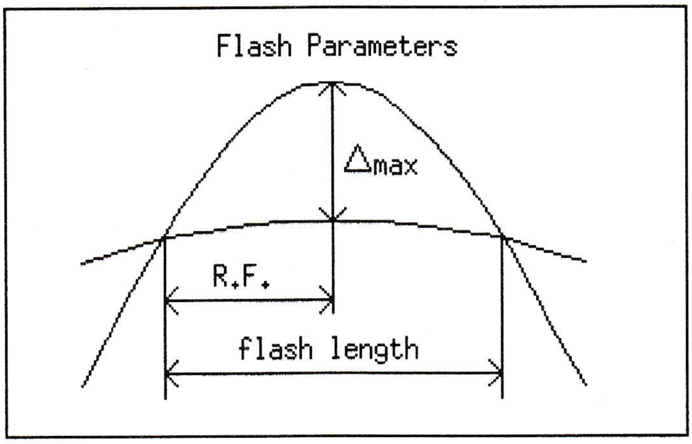

Figure 2: *To determination of flash parameters. Here Δ_{max} is deviation of the flash maximum from the mean level, R.F. is rising front duration, flash length is full length of the flash.*

6. Detection of flashes and determination of their parameters

The development of algorithms of automatic detection of flashes, determination of their parameters and investigation of statistical properties were the next stages after the plotting of the light curve and filling its gaps.

With this end in view, two smoothed curves $< I(t_i) >_k$ and $< I(t_i) >_K$ are plotted on the basis of the light curve with the filled gaps $I(t_i)$ using the sliding average method with a rectangular window:

$$< I(t_i) >_k = \frac{1}{k} \sum_{j=i-k/2}^{i+k/2} x_j, \quad < I(t_i) >_K = \frac{1}{K} \sum_{j=i-K/2}^{i+K/2} x_j,$$

here x_j is the number of counts in the window of the curve. The former curve is plotted with a small smoothing window, $\Delta T_k = k\Delta T$, the size of which can be varied. The size of the smoothing window of the latter curve $\Delta T_K = K\Delta T$ is an order of magnitude larger. This curve "senses" but slightly the flashes, and only those which duration is comparable with the length of its smoothing window. As k diminishes, the first curve more and more faithfully copies the initial curve. The smaller k, and hence K, the shorter flashes can be detected. The difference in level between the former and the latter curves suggests that a flash may possibly be present in the examined region of the light curve.

We say a flash is detected if

$$\delta = \frac{< I(t_i) >_k - < I(t_i) >_K}{\sqrt{S_k^2 + S_K^2 - 2cov < I(t_i) >_k < I(t_i) >_K}} \simeq$$

$$\simeq \frac{< I(t_i) >_k - < I(t_i) >_K}{\sqrt{S_k^2}} = \frac{\Delta_{max}}{\sqrt{S_k^2}} \geq 3.$$

Here S_k^2 is the variance of the quantity $< I(t_i) >_k$, S_K^2 is the variance of the quantity $< I(t_i) >_K$, the summand $2cov < I(t_i) >_k < I(t_i) >_K$ appears because $< I(t_i) >_k$ and $< I(t_i) >_K$ are not mutually independent. The moments of the second order of the distribution of the quantities $< I(t_i) >_k$ and $< I(t_i) >_K$ have the form (Korn, Korn, 1974):

$$S_k^2 = S^2/k, \quad S_K^2 = S^2/K = o(S_k^2)$$

and

$$cov < I(t_i) >_k < I(t_i) >_K = kS^2/kK = S^2/K = o(S_k^2),$$

where S^2 is the sample variance of the data from the comparison star.

The average intensities of the object and the comparison star are assumed to coincide. Thus the quantity δ may serve as the indicator of variability. The duration of the flash is determined as the separation of two points of intersection of the curves considered between which Δ_{max} lies, such that $\delta \geq 3$. In the absence of variability this condition corresponds to a probability of appearance of a flash less than 1 %. The duration of the rising front of the flash is determined as the difference of the flash maximum and its onset (Fig. 2).

For convenience of physical interpretation of the results, the amplitude ΔI of the flash is converted to stellar magnitudes by the formula

$$\Delta m = 2.5 \lg \frac{I_{max} - I_{bg}}{< I > - I_{bg}}.$$

Here $I_{max} =< I > + \Delta I$ is the intensity of radiation in the flash, I_{bg} is the intensity of background radiation, $< I >$ is the average intensity.

As a result of analysis, diagrams "amplitude—flash duration" and "amplitude—rising front duration" and also histograms of the distribution of flashes

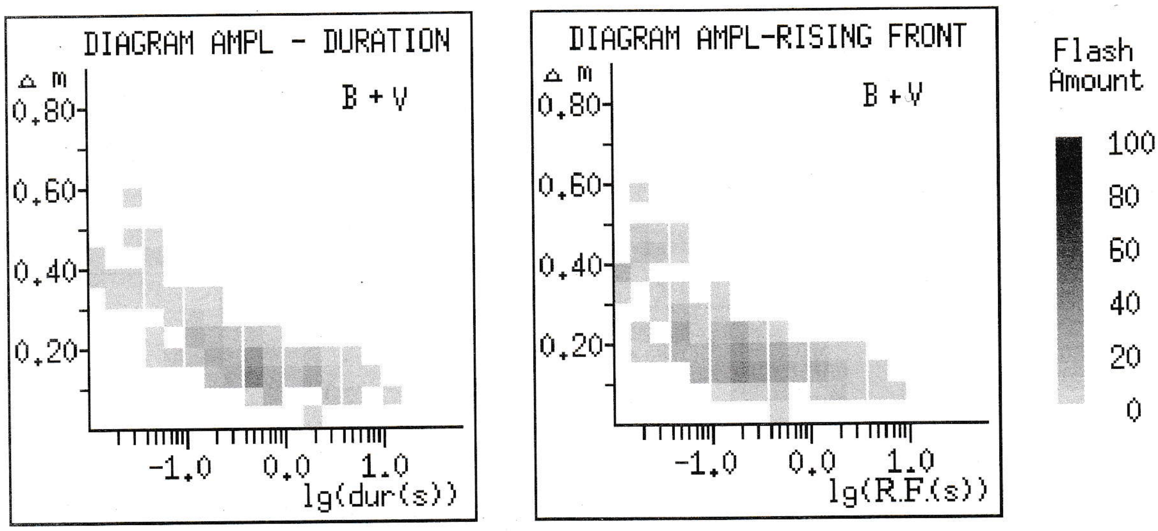

Figure 3: *Dependences of relative stellar magnitudes of Nova Persei flash aggregate observed at the 6 m telescope on 19 January, 1992 on their full length and on rising front length.*

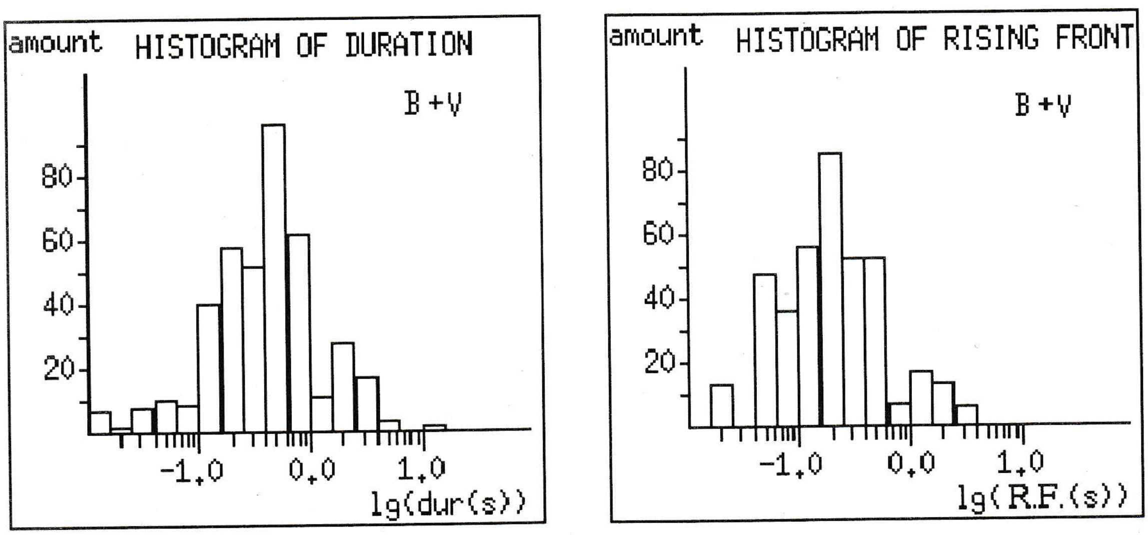

Figure 4: *Histograms of distribution of Nova Persei flashes according to their full duration and to the rising front duration.*

on their durations and rising front durations are constructed. They allow interrelation of parameters of physical phenomena occurring in the star to be established in a summary statistical form. The diagrams and histograms are constructed with subtraction of the distribution of flashes obtained from the comparison star. It may be assumed that excess of the number of flashes in the object over the number of flashes in the comparison star, all other things being equal, is caused by physical processes in the object in question. Not nearly every flash observed in the object may be real. Common parameters of flashes observed with minor excess over the statistical background can be investigated only from their collection.

In the diagrams displayed in Fig. 3 an inverse relationship of the amplitude of flashes to their length and to the length of the rising front can be seen. On the whole, the diagrams are similar, which suggests some symmetry of flashes.

When examining the histograms in Fig. 4, one can notice that flashes of 0.5 s in duration are the most typical. The form of the histograms points to the fact that symmetric flashes predominate.

7. Conclusion

- The examples of reduction of observational data presented in the paper show that the complex of programmes considered allows the data to be generalized and general rules which cannot be displayed by a separate light curve to be revealed.

• The algorithm of filling the gaps by model Poissonian fluxes remains currently applicable since the problem of discontinuity in the acquisition of observational data has not been resolved yet. The light curves derived by the methods described above can further be subjected to spectral analysis, which presented a problem before they were created.

• Despite the fact that the programme was developed specially for reduction of data on Nova Persei, it can be used in studying other objects, such as flare stars.

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