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Magnetic CP stars of the main sequence. 1. Diagnostic techniques for magnetic field

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The first part of a survey dedicated to magnetic CP stars is presented. The initial stage of the study of stellar magnetism is considered and the essential stages in the methods and observational equipment development are shown. Fundamental polarimetry and some applications of the Zeeman effect are set out. Information about ways to calculate the effective Lande factors is adduced. A detailed analysis of different methods of magnetic field measurements is presented. The most widespread methods are: 1) a "photographic" method, which is the search for shifts of orthogonally polarized σ-components of lines; 2) photoelectric measurements of the V Stokes parameter in hydrogen line wings. The following methods are also considered: the Mathys method of momenta, multi-linear methods of spectropolarimetry, in particular, the LSD Donati's method, the Doppler-Zeeman mapping and others. Advantages and disadvantages of each of them are analyzed. Application guidelines are given. Some prospects for the development of magnetic field diagnostics methods are considered.

Key words: stars: chemically peculiar – stars: magnetic fields

1. Introduction

Magnetism is a widespread phenomenon in our Galaxy and beyond. Magnetic fields are responsible for the generation of various bursts, explosions and other manifestations of instability in cosmic objects. The values of measured fields cover a very wide range: microgausses in interstellar matter, milligausses in dense protostellar clouds, units of gauss in the general field of the Sun, kilogausses in local fields of spots on the Sun and Sunlike stars and in the total magnetic field of chemically peculiar stars, megagausses in magnetic white dwarf stars and other degenerate objects.

The study of mechanisms of cosmic magnetic field generation and support and their role in evolution of stars and galaxies is one of the most important directions in modern astrophysics. With the exception of lucky accidental discoveries, there are two regular ways to obtain new knowledge in the experimental astrophysics: 1) the implementation of more and more powerful telescopes, sensitive and precise light detectors, refined processing methods; 2) long observational series with the same equipment for detecting regularities in the behavior of a specified object.

Of course, the indicated ways can co-exist simultaneously, but an experience has shown that because of the strong competition in the observational time allocation at big instruments, the second way is rarely realized, only in the case of observations of unique objects. And as a rule, the radically new results are a consequence of development and implementation of new methods.

A prominent example supporting this consideration is a history of the study of cosmic magnetism. Before the creation of special devices — polarization analyzers — the magnetic fields outside the Earth were not known at all! And after the development of methods for magnetic measurements of stellar objects, which led to the detection of magnetic field in sunspots by Hale in 1908 and in stars by Babcock in 1947, a new science — cosmic electrodynamics — appeared.

The timeliness of stellar magnetic field studies is also confirmed by the fact that all the biggest telescopes becoming operational were equipped with devices for measurement of the Zeeman effect. It was so in the 50-s and 60-s of the 20^{th} century (the 5 m telescope of the Palomar observatory and the 3 m telescope of the Lick observatory), in the 70-s and 80-s, when the magnetic studies were started at the 6 m SAO telescope and the 3.6 m ESO telescope (Chile). In the 21^{st} century the Zeeman observations are included in programs of the largest 8 m ESO VLT telescopes.

In the 20^{th} century the concepts of the magnetic field role in astrophysical processes were changing cardinally — from the total neglect to overestimation of its action. A phrase like "...it can be a consequence of the magnetic field effect" was often met in papers which could not explain some phenomena by any known mechanisms. By the felicitous remark of a patriarch of the cosmic magnetism exploration Leon Mestel (2001) concerning the role of magnetic fields: "once likened magnetic fields in astronomy to sex in psychology. Before Freud, it was ignored; then immediately after Freud, it was going to explain everything; but with time, a more balanced view has evolved.. Hopefully, ... in astrophysics will continue the development of this more balanced view".

Astronomers developed various research techniques and obtained reliable proof of existence of large-scale magnetic fields in our Galaxy, total and local fields in stars of various types. The most elaborated method of studying the stellar magnetism is an analysis of the Zeeman effect manifestations in lines by spectropolarimetry techniques. In practice other methods are used when the analysis of the Zeeman effect manifestations is impossible for one reason or another.

The chemically peculiar (CP) stars are very suitable objects for studying cosmic magnetism: having general global highly-intensive fields they are bright enough to allow very detailed and high-precision investigation of their spectra with big telescopes. In its turn it gives an opportunity to discover various regularities in the complicated process of interaction between plasma and magnetic field.

On the other hand, CP stars have unique paradoxical features. Whereas the magnetic fields on the Sun and in many other objects cause explosions and bursts, in the case of CP stars they play a stabilizing role: to explain the chemical peculiarity it is necessary that in a stable non-turbulent atmosphere there would operate a diffusion mechanism that could create during million of years the observed spots and other anomalies of chemical abundance. And the atmosphere stability is to be provided by magnetic field!

In spite of more than a half-century history of research, the CP stars are still poorly understood objects. First of all, the problem of origin of their large-scale fields is not solved yet. Theoreticians proposed diverse explanations, but two competitive hypotheses are the most elaborated: 1) the magnetic field is relic, it was formed from the interstellar medium together with the star; 2) a dynamo mechanism operates in convective nuclei of CP stars; the generated field is carried out on a surface and becomes observable.

The real picture of the CP-star magnetic field origin and its subsequent evolution can be achieved by observational tests of consequences of alternative mechanisms (the relic one and the dynamo in nucleus). To solve this difficult problem, it is necessary to observe magnetic fields of large number of CP stars with various masses, temperatures, rotational velocities, ages and spatial distributions in the Galaxy.

Magnetic fields of CP stars can be measured only with very big telescopes. The methods of observation and data processing must provide the system stability and correspondence to common standards. With this end in view an equipment complex was elaborated in SAO RAS under the direction of Prof. Yu.V.Glagolevskij. Diverse devices, which were often unique ones, allowed the domestic researchers to make unique measurements of stellar magnetic fields with the 6 m telescope. During a quarter of a century an extensive material was obtained with the BTA: about 30% of magnetic observations of CP stars all over the world in 1977–2004; about 40% of all magnetic stars newly discovered in that period. About a quarter of all magnetic curves were constructed. The results obtained with the 6 m telescope are widely known. SAO RAS is one of the world centers for the study of stellar magnetism.

2. The initial stage of stellar magnetism study

2.1. The discovery of a phenomenon of spectral line splitting in outer magnetic fields. Some definitions

The Zeeman effect — a phenomenon of spectral line splitting under action of an outer magnetic field on emitting matter — was named after its discoverer, a Dutch physicist Peter Zeeman.

In 1896 P.Zeeman set a burner with the sodium flame between electromagnet poles and discovered that when a sufficiently strong field was turned on, the sodium D line (a 5890 and 5896 Å doublet which merges at low resolution) was broadened, the line ends being polarized. The Zeeman effect was first explained by H.Lorentz in 1897 within the framework of the classical theory.

Zeeman did not see the line splitting. An important point was that the wings of a broadened line were polarized in accordance with the classical theory (see, e.g., Shpolsky 1984). If he had used a stronger field or the equipment of higher spectral resolution, he would have seen the splitting in the case of sodium, but the picture would be more complicated than a simple triplet.

It turned out that the visible picture of splitting depends on the position of an observer with respect to the direction of magnetic field. When the observations are made perpendicularly to the field (a transversal effect) all line components are polarized linearly: a part is parallel to the field (the π-components), another part is perpendicular (the σ-components). When the observations are made along the field (a longitudinal effect) only the σ-components are visible, but their linear polarization is replaced by circular. The intensity distribution in the system of components is complex.

The Zeeman effect observed in absorption spectra is known as the inverse one: all its regularities are analogous to regularities of the direct effect (observed in emission lines), but with the opposite sign. If at the transversal effect the line is split into three components this is the normal Zeeman effect, and the splitting into more components is called the anomalous Zeeman effect. It was properly explained by the quantum theory.

In the case of the anomalous effect in the atomic spectral lines a group of π-components is observed instead of the non-shifted π-component, and a group of σ-components is observed instead of each σ-component, the pattern of splitting being symmetric about the initially non-split line. Distances between the neighboring π-components and the neighboring σ-components are equal. Their total number can reach several tens.

In 1912 Pashen and Back discovered an important and interesting phenomenon. It turns out that in very strong fields the complex Zeeman effect becomes simple again: a complicated pattern of splitting is replaced by the simple Lorentz triplet. This effect is called a magneto-optical transformation or the Pashen-Back effect. In this case the field is said to be strong if it causes a splitting which far exceeds the natural multiplet splitting. In the magnetic field of the order of millions of gauss the quadratic Zeeman effect becomes essential.

2.2. The discovery of magnetic field on the Sun

The study of cosmic magnetism was initiated by George E. Hale who discovered magnetic field of the sunspots in 1908. In 1892, before Zeeman's discovery, Young had noticed that the lines, which are narrow and sharp in regions without spots, are broadened in the sunspots and some of them are even split into two lines.

In 1908 Hale decided to test if this was a manifestation of the Zeeman effect and set a Fresnel rhomb and Nicol prism in front of the slit of the tower solar telescope spectrograph. Here is how he describes this discovery (Hale 1908): “On June 25 I obtained some good photographs, in the third order, of the region $\lambda 6000-6200$... These clearly showed a reversal of the relative intensities of the components of spot doublets when the Nicol was turned through an angle of 90°. Moreover, many of the widened lines were shifted in position by rotation of the Nicol, indicating that light from the edges of these lines is circularly polarized in opposite directions. The displacement of the widened lines appeared to be precisely similar in character to those detected by Zeeman in his first observations of radiation in a magnetic field”. The value of magnetic field in the sunspots was of the order of several kilogauss.

Note that earlier Hale was also trying to detect magnetic fields on the Sun. As early as in 1905 he unsuccessfully attempted to determine visually linear polarization as a consequence of the Zeeman effect. Then he decided to use a photographic plate (see the details in the perfect paper by Stepien (1997)). Thus, the application of new equipment permitted magnetic fields in the sunspots to be discovered.

Later on, Hale (1913) tried to detect the total magnetic field of the Sun using one of the most perfect devices of that time — the 75-feet spectrograph of the tower solar telescope. He stated that it was difficult to measure the Zeeman shift, but nevertheless he assumed that the magnetic field at the Sun pole was 50 G. The stronger lines gave fewer measured shifts than the weak ones. At that time it was already known that the strong lines originate in higher atmosphere layers, so Hale interpreted such a relationship as a decrease of field with height.

During more than 40 years astronomers thought that the total field of the Sun is 50 G. Even in 1953 Tissen detected its value to be 53 ± 12 G under the assumption that it is a dipole one. Only in 1953 Babcock developed a photoelectric method and did detected a very weak (units of a gauss) total magnetic field of the Sun. He considered that the error of Hale and other authors was caused by the fact that they had to measure very small shifts of lines that noticeably differ in intensity.

Thus, the history of detection and further study of the magnetic field of the Sun is very instructive: whereas the strong field in spots was measured correctly right after its detection, the total weak field was beyond measurements for a long time. Its value and structure were a steady fallacy for decades.

2.3. Detection of magnetic fields of stars

As early as in 1915, Hale considered a possibility to detect magnetic field of stars and began designing equipment fit for such investigations, but this work received no further development (see Babcock 1967).

In 1937 Minnaert published a paper “On the possibility of detecting the general magnetic field of a star” (Minnaert 1937). If the magnetic field of stars is equal to the total field of the Sun, then it causes the line shifts (~0.001Å) defying measurement, but if it is 10–20 times stronger than the solar one, then the doublets with circular polarized components should be looked for in high-resolution spectra. As in the classical Hale experiment, he suggested to set a Nicol prism and a quarter-wave plate oriented at an angle of 45°. When turning the plate through an angle of 90° we obtain a series of spectra in which the lines are shifted to the right or to the left. The measurements are differential; therefore a high precision could be expected. It is easier to measure the shift of sharp lines when observing a star with the axis of rotation directed towards us.

If Minnaert had succeeded in inventing a method of obtaining left- and right-circular polarized spectra simultaneously, he would have been a discoverer of stellar magnetism.

But the magnetic field of stars was detected only 9 years later, in 1946, by a prominent astronomer Horacio Babcock (1947a,b), who determined a correct value of the total magnetic field of the Sun in 1953. He was animated by the popular assumption (Minnaert 1937) that the magnetic field is generated by stars, and the magnetic intensity is proportional to the velocity of rotation at the equator. Comparing the magnetic field of the Sun (then 50 G, the wrong value) and $v \sin i = 2$ km/s with the rotation velocity of B, A, F stars ($v \sin i = 60$ km/s) he obtained that their magnetic field can reach 1500 G.

Babcock made a number of simplifying assumptions: the axis of rotation and the magnetic axis coincide and are directed right to an observer; the star is a sphere; the darkening coefficient is equal to 0.45; the equivalent widths of absorption lines are invariable along the star hemisphere projection directed toward an observer; the magnetic field distribution is similar to the magnetic field distribution of a uniformly magnetized sphere. Taking into account the Seares (1913) formulae, he decided to observe the A stars with narrow and sharp lines. In the case where the star is seen from its pole, the integral Zeeman effect has a preferred direction and the line components after passing an analyzer of left and right circular polarization are shifted in opposite directions proportionally to the field strength.

Babcock developed and produced a differential analyzer of circular polarization. The schematic diagram of this device (a quarter-wave plate + a calcite block) has been used to our days. This analyzer was used to simultaneously record two opposite circularly polarized spectra on a photographic plate, which gave a very good accuracy when using a high-precision spectrograph. The field measurement reduced to determination of shifts between the left and right line sigma-components visible separately in each polarization. It was found that the shift of the center of gravity of the Zeeman components $\Delta\lambda = \pm 0.311\,\mathrm{B}$, i.e. the integral effect all over the star is 31% of the effect which would occur if we could separate the light from the magnetic pole of the star. The very first star observed by him with the analyzer (78 Vir with anomalous metal lines) showed the field on the pole to be as large as 1500 G, which was in perfect agreement with the preliminary assumptions!

Babcock's early papers (1947a,b) contained the value of the so-called polar field, B_p, and the relation between the magnetic intensity near the star pole and the component shift $\Delta\lambda$ (in Å) of the normal triplet observed with the help of the circular polarization analyzer:

$$\Delta\lambda = 1.45 \cdot 10^{-13} \lambda^2 g B_p\,. \qquad (1)$$

However, later on he used a notion of the effective magnetic field B_e, which is equal to 0.311 B_p.

In that case the shift for a normal triplet (in Å) is

$$\Delta\lambda = 4.67 \cdot 10^{-13} \lambda^2 g B_e\,, \qquad (2)$$

where g is the Lande factor of the corresponding energy-level transition, which affects the number of components and relative distance between them, their intensities and polarizations, whereas the picture scale is proportional to magnetic field.

In spite of the fact that the Zeeman splitting grows as λ^2, in practice stellar magnetic fields were measured mostly in the blue spectral range. The number of convenient metal lines is smaller in the yellow and red spectral bands. Besides, the photographic emulsions were grainier and less sensitive. Later, with the evolution of digital light detectors (photomultipliers and CCDs) a lot of measurements were fulfilled in the red band also.

The history showed that Babcock's choice of objects for search of stellar magnetism was exclusively fortunate: regular large fields with an ordered large-scale structure were detected only for Ap and Bp stars. In the middle of the 20^{th} century, with the methods he used it was possible to detect such fields only for these stars. It was impossible to find magnetic fields of complex structure (similar to solar) with the only photographic equipment existing at that time.

3. Some general questions of spectropolarimetry. The Zeeman effect

3.1. Introduction

The magnetic field values can vary within 20 orders: from 10^{-6} G in interstellar medium to 10^{12}-10^{14} G and more in neutron stars and magnetars. The study of so different values demands application of different methods.

Gnedin and Natsvlishvili (2000) thoroughly reviewed the available methods of study of cosmic magnetic fields. The reader can see the details in this review, and I mention only the essentials: there are direct and indirect methods.

The main and commonly used direct method is the study of the Zeeman effect in its different manifestations, which permits reliable detection and measurement of magnetic fields. Indirect methods of studying the Faraday rotation and the spectrum of linear polarization, effects of stellar activity in separate chromosphere lines, thermal and nonthermal radio emission, etc., are substantially model dependent, therefore, the interpretation of measurement results can be ambiguous.

The study of magnetoactive plasma polarization is of primary importance for astrophysics. A fundamental monograph of Russian researchers (Dolginov et al. 1979) is a major contribution into development of this direction.

3.2. Basic definitions of polarimetry

3.2.1. Mechanisms of polarization origin

The basic physical mechanisms responsible for the origin of polarization of light in astrophysical conditions are summarized in Table 1. The information is compiled from different literature sources (mainly, from a monograph by Dolginov et al. (1979), a review by Gnedin and Natsvlishvili (2000) and some others).

As is seen from Table 1, the reasons for the origin of polarization may be very different. In general, its value is small and, consequently, it can be determined in experiments with low signal-to-noise ratio.

3.2.2. Stokes parameters

In the general case, the polarized state of light is described by a Stokes vector consisting of 4 elements (I, Q, U, V). The first Stokes vector I is the total intensity of the ray, the second and the third parameters Q and U define the intensity of linearly polarized light in different planes, and the forth parameter V describes the intensity of the circularly polarized component. If there is a non-zero V value together with a non-zero value of at least one of the U and Q parameters, then the light is elliptically polarized.

The Q value corresponds to the intensity difference of the orthogonal oscillations azimuths of which form the coordinate frame for the vector. In stellar polarimetry it is common practice that the positive Q value corresponds to oscillations in the north–south direction, whereas the negative Q value does in the azimuth of the west-east direction. The U value corresponds to the intensity difference of the orthogonal oscillations with azimuths turned by 45° northward through the east with respect to the direction of positive Q. The V value is the intensity difference between the right-circular and left-circular polarized components.

For partially polarized light, which is a system of incoherent light waves in different polarization states, the polarizations themselves cannot be summed up since the electrical intensities are non-additive for incoherent light beams. That is why here it is necessary to use a set of 4 Stokes parameters which are additive for them.

In practice the normalized parameters are usually used:

$$q = Q/I, \quad u = U/I, \quad v = V/I,$$

The linear polarization is defined as:

$$p = \sqrt{(q^2 + u^2)}, \quad p = \sqrt{(Q^2 + U^2)}/I.$$

The value of circular polarization corresponds to

$$v = V/I\,.$$

The angle θ between the major ellipse axis along the direction of I and the direction of the north celestial pole determines the position angle of the polarization plane according to the relation:

$$tg2\theta = U/Q\ .$$

Observations are recorded with an optical system which is placed in front of the spectrograph entrance slit and operates as a polaroid. To measure one of the Stokes parameters describing the linear polarization (say, Q) it is necessary to obtain the spectral line profiles in two mutually perpendicular positions of the polaroid. The difference of these two profiles gives the Q profile.

To measure the profile of the Stokes parameter U it is necessary to turn the polaroid axis by 45° to the initial position and to repeat both observations. That is why the time necessary to obtain the linear polarization profiles is twice that for obtaining the circular polarization profile and four times as long as

Table 1: *Polarization of astronomical objects*

Mechanism of origin	**Manifestation in astrophysical objects**
Reflection from a solid surface	The Moon, inner planets and asteroids
Light scattering on small particles	Zodiacal light
Light scattering on molecules (Rayleigh scattering)	Jupiter and outer planets, Venus, late-type stars
Light scattering by free electrons (Thomson scattering	Solar corona, envelopes of early-type stars
The Hanle effect (resonance scattering of bound electrons in magnetic fields)	Linear polarization in emission lines in solar chromosphere and corona
The Zeeman effect	Solar spots, magnetic stars and white dwarfs (circular and linear polarization in spectral lines), radio-frequency emission lines of neutral hydrogen molecules
"Gray-body" magnetoemission	White dwarfs (circular and linear polarization in continuum)
Gyroresonance emission (magnetic bremsstrahlung)	Solar chromosphere and corona
Synchrotron emission (in some cases an inverse Compton scattering or electrostatic bremsstrahlung)	Decimetric radio emission of Jupiter, the Crab nebula, pulsars, radio emission of the Galaxy corona, radio galaxies, quasars

for the intensity profile. Besides, the linear polarization profiles contain much more details and have, on average, a smaller amplitude than the circular polarization profiles. This imposes stringent requirements on the precision and spectral resolution.

The wider use of the circular polarization in the study of magnetic field geometry is also caused by the fact that the Stokes parameter V is invariant relative to the analyzer axis rotation in the picture plane. A different situation arises with the Stokes parameters Q and U depending largely on the polaroid axis position angle α, which leads to considerable technical difficulties with rotating fields of view. Even an exact knowledge of the position angle does not solve the problem entirely since the optimal value of α is not known. When this angle is chosen poorly, the linear polarization amplitude may prove be at the observation noise level. To avoid this situation, it is necessary to register the Q and U profiles with different polaroid angles α covering the range from 0° to 90°. Even Babcock (1953) noted that the implementation of this method consuming a lot of observational time is possible only with the largest telescopes and for the brightest stars.

Stepanov and Severny (1962) suggested a photoelectric method for measuring the value and direction of the solar field described by the angle γ of the field vector inclination towards the line of sight and the azimuth θ of the transversal field component, which gives the absolute value and direction of the field vector.

3.3. Transfer of polarized emission

Diagnostics of magnetic field properties in stellar atmospheres is not a simple problem. In most cases some values describing certain spectral lines are measured and then the field properties are "drawn" from them with the help of different models of formation of lines. It is seen from literature that the number of accepted assumptions is very substantial.

Practically in all cases analysis of magnetic fields in non-degenerate stars is based on measuring the Zeeman splitting of spectral lines. Three possibilities should be taken into consideration: 1) in a magnetic field of the value of B the lines are split into the π- and σ-components with the division proportional to B; 2) the transversal field leads to the orthogonal linear polarization of the π- and σ-components parallel and perpendicular to the field; 3) the longitudinal field leads only to the σ-components with opposite circular polarizations. The properties of spectral lines depend on the joint effect of the Zeeman splitting and the transfer of polarized emission.

The first solution of the transfer equation in the presence of a magnetic field for stellar atmospheres was devised by Unno (1956). In my review I will dwell only on the key aspects of the theory. I will pay more attention to papers by Russian astronomers: as a rule they are insufficiently presented in English-language publications and are not cited in foreign reviews in full measure. First of all, I mention papers by representatives of the Leningrad school: Yu.N.Gnedin, A.Z.Dolginov, N.A.Silantiev, G.G.Pavlov and others. Basic results of studying the transfer equation for polarized emission in anisotropic medium were generalized in the fundamental monograph by Dolginov et al. (1979) (its second edition was published in English in 1996 (Dolginov et al. 1996)).

The papers by D.N. Rachkovsky (e.g. 1972, 1974) made a certain contribution to understanding the processes of line formation in magnetic field at early stages of studying stellar magnetism.

V.L. Khokhlova has founded a new research trend in which the refined investigations of atmospheres of magnetic stars are carried out by numerical techniques of applied mathematics. Solving the inverse problem by means of local line profiles, Khokhlova and her colleagues mapped the distributions of elements and magnetic field over the star surface. The local profiles are the result of emission transfer in every point of the photosphere surface and depend on the local abundance, the value of the magnetic field and local physical conditions. The basic results are summarized in a review by Khokhlova (1983). Later on, this research trend was substantially developed by N.E. Piskunov theoretically and by T.A.Ryabchikova practically.

Landi degl' Innocenti (1983) suggested a general formula of the transfer equation for polarized light. He accepted a number of simplifying hypotheses, in particular, the atmosphere is regarded as being plane-parallel and static. The lines are formed by radiation averaged over the whole visible stellar surface, the center accounting for a considerably greater weight.

Hereinafter I will use the approach applied by Landstreet (2001). To understand the idea how spectral lines change due to magnetic field, it is useful to consider solutions of 4 equations of polarized transfer for the I (intensity), Q and U (linear polarization) and V (circular polarization) Stokes parameters in the case of the Milne-Eddington atmosphere (the linear function of a source, a constant ratio of opacity in the line to continuum) (Mathys, 1989).

In case of weak lines or weak fields the solutions are simple. The following fundamental assumptions are used in most magnetic field measurement methods: 1) the average separation of circularly polarized line centroids $(I+V)$ and $(I-V)$ is proportional to z$\langle B_z \rangle$ (condition of a weak field), where z is the average Lande factor, $\langle B_z \rangle$ is the line-of-sight-averaged component of the magnetic field; 2) in a weak field approximation $V \sim z\langle B_z \rangle \, dI/d\lambda$ in the local line (in case of a small broadening *v sin i*); 3) the separation of resolved σ-components is proportional to $z\langle B_z \rangle$ (the condition of a weak field). These equations are basic when the field is measured as moments (Mathys 1989).

3.4. Principles of the theory of the Zeeman effect

3.4.1. An atom in magnetic field

The quantum theory gives a complete explanation of the Zeeman effect. When the magnetic field is weak (when the L–S coupling is assumed), there are two types of the Zeeman splitting: a normal effect with the triplet splitting of spectral lines and an anomalous effect when the line is split into a lot of π- and σ-components. In a very strong magnetic field (when the L–S coupling is violated) the Pashen-Back and quadratic Zeeman effects are observed.

In the presence of magnetic field the atom levels are splitted into (2J+1) sublevels each being described by its magnetic quantum number M that varies between J and –J, where J is the quantum number describing the total atom momentum.

Such a description is suitable for a weak magnetic field on the assumption that the magnetic splitting of levels is small in comparison with the fine structure, i.e. the separation of the lines of one term occurs.

3.4.2. The normal Zeeman effect

The normal Zeeman effect is realized only for singlet lines. In a magnetic field the singlet atom levels (spin S = 0) are split, whereupon each line of the frequency ν is split into three components: the π-component with the same frequency ν and two σ-components with frequencies

$$\nu = \nu_0 \pm 1.4 \cdot 10^6 B, \qquad (3)$$

where ν is the frequency in Hz, B is the magnetic field in gauss. All line components are elliptically polarized.

In observations along the magnetic lines of force (the longitudinal Zeeman effect) the central π-component is not seen, and the two σ-components of equal intensity are circularly polarized in opposite directions. In observations taken perpendicular to the field (the transversal Zeeman effect) all three components are polarized linearly: the π-component is parallel and the two σ-components are perpendicular to the field direction. In this case the splitting of lines is small: about 0.01Å in a magnetic field of 1000 G for the spectral range with the center at 5000 Å.

3.4.3. The anomalous Zeeman effect

The overwhelming majority (more than 80%) of lines in stellar spectra show an anomalous Zeeman effect

(Romanyuk 1984).

In the general case of non-singlet lines in the L–S coupling of spin $S \neq 0$ in a magnetic field every atom level is split into 2J+1 sublevels with the shift energies

$$E_M = E_0 \pm 9.27 \cdot 10^{-21} B g M_j, \tag{4}$$

where E_0 is the level energy in the absence of a field (erg), B is the magnetic field in G, g is the Lande factor of the level, M_j is the J projection onto the direction B.

The Lande factor g for each level is determined as:

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}, \tag{5}$$

where L and S are orbital and spin momenta, J is the total angular momentum.

When the effect is anomalous the spectral line may be split into several tens of π- and σ-components. The splitting pattern is totally determined by the configuration of electrons in the atom and is explained by the vector atom model in the quantum theory.

At a first approximation, the polarization of groups of π- and σ-components is the same as that of corresponding components in the normal effect. As Back and Lande showed (1925), there are only six types of the Zeeman pattern. Their properties are summarized in Table 2 (taken from Mathys 1989).

The seventh type is the Zeeman triplet arising between two levels with equal Lande factors or between the levels with J = 0. Thus, the normal effect is a special case of the anomalous Zeeman effect.

Another peculiar case is the zero magnetic transitions. They arise between two levels with the non-zero Lande factor or between the level with the zero factor and the level with J = 0. Zero magnetic lines can be very useful in astrophysical investigations: their profiles are not distorted by the Zeeman effect, and so, the correct parameters of the magnetic star atmospheres can be obtained.

The following properties are common for all Zeeman patterns irrespective of their type:

- for all transitions the Zeeman patterns are symmetric about the line center;
- π-components are symmetric about the line center;
- to a first approximation the Zeeman splitting can be described by one parameter — an effective Lande factor z, the shift in wavelengths of the centers of gravity of σ-components expressed in units of $\Delta\lambda$.

3.4.4. The Pashen-Back effect

In very strong fields (> 10 kG), when magnetic splitting becomes larger than multiplet splitting, the anomalous Zeeman effect begins turning into the normal one. This phenomenon known as the Pashen-Back effect is a result of the fact that the L–S coupling is violated in a strong magnetic field.

In this case the energy E_M of a level shift from the center of the initial multiplet structure (in ergs) is:

$$E_M = E_0 \pm 9.27 \cdot 10^{-21} (M_L + 2M_S) B, \tag{6}$$

where M_L and M_S are the orbital and spin quantum numbers, respectively.

The Pashen-Back effect for different lines occurs at different magnetic fields depending on the multiplet type. As a rule, when a field is stronger than 10 kG, the influence of the partial Pashen-Back effect should be taken into account (Mathys and Stenflo 1987 a, b; Mathys 1990). The basic features are as follows:

- whereas the line formed under the simple Zeeman effect has the same wavelength as in the absence of field, the lines formed under the partial Pashen-Back effect have the common shift;
- the lines formed under the partial Pashen-Back effect are not symmetric about their centers in contrast to the simple Zeeman effect;
- under the partial Pashen-Back effect the total intensity of lines differs from that in the absence of magnetic field.

3.4.5. Quadratic Zeeman effect and a hyperfine structure

The real measurements of the quadratic Zeeman effect can be expected in superstrong magnetic fields (10^6–10^8 G) of white dwarfs and neutron stars as a shift of lines toward the short wavelengths. For hydrogen this shift can be calculated by the following formula:

$$\Delta\lambda = -4.98 \cdot 10^{-23} n^4 \lambda^2 (1 + M^2) B^2, \tag{7}$$

where n is the main effective quantum number, M is the magnetic quantum number, B is the magnetic field in G, λ and $\Delta\lambda$ are expressed in angstroms.

In spectra of magnetic CP stars with fields weaker than 100 kG the influence of this effect is reduced to the shift of lines toward the violet spectral region by a negligible quantity ($\ll 10^{-5}$ Å).

In a number of situations the hyperfine structure can affect, say, the lines of rare earths. This question was studied in details by Landi degl' Innocenti (1975). The conclusion is that the hyperfine structure

Table 2: *The types of the Zeeman pattern of splitting*

Type	**Multiplicity of levels**	$\pi-$ **component**	$\sigma-$ **component**	**(J1-J2)(g1-g2)**
1	Odd	Odd number, the central component is the strongest one	Odd number, intensifying towards the center	< 0
2	Odd	Odd number, the central component is the strongest one	Odd number, weakening towards the center	> 0
3	Odd	Odd number, the central component is the weakest one	Even number, symmetrical to its center of gravity, the central component is the strongest one	$=0$
4	Even	Even number, the central component is the strongest one	Even number, intensifying towards the center	< 0
5	Even	Even number, the central component is the strongest one	Even number, weakening towards the center	> 0
6	Even	Even number, the central component is the weakest one	Odd number, symmetrical to its center of gravity, the central component is the strongest one	$=0$

does not introduce any additional difficulties in comparison to the pure Zeeman effect. It can influence the sharpness of spectral lines of some elements.

3.5. Calculations and determinations of the effective Lande factors

3.5.1. Calculations of the Lande factors

In most cases the direct observation of line splitting in stellar spectra fails. Only the total shift of π- and σ-components or their groups is visible (in the anomalous effect). That is why it is important to calculate the effective Lande factor z for a line as a weighted average parameter for the groups of π- and σ-components that characterizes the line sensitivity to magnetic field on the whole.

The Lande factor can be calculated or determined experimentally. Extensive lists of laboratory measurements of the factor are adduced in the paper (Martin et al. 1978). I recommend a fundamental review by Mathys (1989) on this subject.

Unfortunately, the laboratory determinations are fulfilled for a relatively small part of spectral lines of astrophysical interest, therefore one has to use the calculated factors. There are cases of very big differences between experimental and calculated values, sometimes twice and more.

At present extensive calculations are carried out. Beckers' tables (1969) are used most often. The effective Lande factors in them are calculated for multiplets under the assumption of the L–R coupling in the following way:

$$z = \bar{g} + \frac{\Delta J \Delta g (2\bar{J}+1)}{4}, \quad \bar{g} = \frac{g_u+g_l}{2}, \quad \bar{J} = \frac{J_u+J_l}{2}, \quad \Delta g = g_u - g_l, \quad \Delta J = J_u - J_l \tag{8}$$

where g_u and g_l are the Lande factors for upper and lower levels, respectively, other designations are the same as in formula (5).

The Lande factors in the VALD (Vienna Atomic Line Database) are calculated on the basis of Beckers' tables (Piskunov et al. 1995, Kupka et al. 1999).

Earlier, with the help of these tables, I compiled a list of the Lande factors for 830 lines that are predominant in stellar spectra (Romanyuk 1984).

Discussion of the problem of determination of the Lande factors when the L–S coupling is violated will be found in the literature of the subject. For example, the formula for calculations in the case of $J-1$ couplings is presented in the paper by Mathys (1990).

Kurucz (1979, 1992, 1993) uses the experimental values of the Lande factors and, if they are absent, the ones determined from detailed calculations of atomic parameters. Mathys (1990) compared the factors used by Kurucz and those calculated by simple formulas (5, 8). He noted that some of the calculated values are very inaccurate and, in particular, much less accurate than most experimental data.

3.5.2. Magnetosensitive lines

The effective Lande factors z for more than 95% of all measured spectral lines in spectra of magnetic stars are within a range of 0.5–2.0 (in units of the Bohr magneton). Certainly, the lines with large factors are preferable for magnetic measurements, but usually they are weak. Nevertheless, they may be useful in the study of fine effects, which cannot be exhibited in lines with lower magnetic sensitivity.

I compiled a list of lines with the Lande factors $z > 3$ (Romanyuk 1984). It is given in Table 3. Moore (1945) and Beckers' (1969) tables were used as the sources. The neutral iron line 4210.35 Å which is well visible in spectra of magnetic stars is the strongest of them. The line FeI 5250.2 Å, which holds the second place in intensity is widely used in measurements of solar magnetic fields since it is intensive enough in spectra of cool stars.

3.5.3. Lines insensitive to magnetic field

The lines that are insensitive to magnetic field and are not distorted by it are used for determination of magnetic star atmosphere parameters and the value v sin i. Lists of zero lines being of interest for astrophysics are presented in the papers by Adelman (1973) and Landstreet (1969). Unfortunately, these lists contain very few lines suitable for measuring; therefore I considered it possible to add the lines that are affected by field, but weakly. The best known lines with a small (or zero) Lande factor are listed in Table 4. The sources are the same as used when compiling Table 3.

Note that in practice the strong line FeII λ4508.28 Å is used.

In conclusion note that the general description of the Zeeman pattern is a big statistical work since a huge number of lines must be taken into consideration. The necessary formulas are derived in the fundamental paper by Mathys (1989).

All aforesaid concerns the case of weak field, where the quadratic Zeeman effect is inessential and the magnetic splitting of levels is small in comparison with the multiplex structure of corresponding terms. However, starting with fields of 10 kG, the atom level shifts should be analyzed with due account for the partial Pashen-Back effect (Mathys and Stenflo 1987a; 1990).

Table 3: *Lines with the effective Lande factor* $z \geq 3$

λ	element	multiplet	z	lg gf
3119.3	Gd II	(10)	3.00	—
3139.6	Fe I	(161)	3.00	−4.63
3158.2	Fe I	(160)	3.25	−3.55
3162.3	Fe I	(159)	3.17	—
3175.9	Fe I	(333)	3.00	−1.87
3221.9	Fe I	(156)	3.00	−0.69
3228.9	Fe I	(157)	3.00	−3.39
3390.8	Gd II	(73)	3.67	−0.42
3417.3	Gd II	(91)	3.33	−0.10
3417.3	Gd II	(91)	3.33	−0.10
3462.9	Mn II	(12)	3.00	−1.25
3598.9	Fe I	(322)	3.00	−1.83
3815.5	V I	(28)	3.33	−1.46
3867.2	Gd II	(50)	3.00	−1.20
3908.9	Fe I	(153)	3.00	−4.58
3966.4	Fe II	(3)	3.00	−6.87
4070.2	Mn I	(5)	3.33	−1.09
4080.8	Fe I	(557)	3.00	—
4116.6	V I	(27)	3.33	−0.82
4210.3	Fe I	(152)	3.00	−1.05
4327.1	Gd II	(—)	3.00	+0.04
4558.0	Gd II	(44)	3.20	−1.54
4654.7	Cr I	(186)	3.00	−0.81
4878.2	Fe I	(318)	3.00	−1.07
5175.8	Gd II	(114)	4.00	—
5250.2	Fe I	()	3.00	−4.84
5986.5	Fe II	(24)	3.00	−6.23
6258.5	V I	(19)	3.33	—

Table 4: *Lines with low sensitivity to magnetic field*

λ	element	multiplet	z	lg gf
3402.43	Cr II	(21)	0.00	−1.15
3533.20	Fe I	(326)	0.50	−0.12
3608.86	Fe I	(23)	0.50	+0.02
3745.90	Fe I	(5)	0.00	−1.41
3767.19	Fe I	(21)	0.00	−0.31
3849.97	Fe I	(20)	0.00	−0.86
4064.35	Ti II	(106)	0.50	−1.68
4284.21	Cr II	(31)	0.50	−2.25
4425.44	Ca I	(4)	0.50	−0.18
4443.20	Fe I	(350)	0.50	−0.95
4464.45	Ti II	(40)	0.33	−1.62
4491.40	Fe II	(37)	0.40	−2.89
4508.28	Fe II	(38)	0.50	−2.41
4634.11	Cr II	(44)	0.50	−1.25
5334.88	Cr II	(43)	0.40	−1.79
5434.52	Fe I	(15)	0.00	−2.19
6239.77	Cr II	(105)	0.00	−3.16

To determine weak fields in the Sun and, recently, in stars, besides the Zeeman effect, the measurements of the Hanle effect, which is concerned with the properties of line profile polarization, are used. It can be used the most effectively in estimating fields of order 10–100 G (see the review by Gnedin and Natsvlishvili, 1997).

4. Diagnostic techniques for magnetic field of CP stars

4.1. Possibilities of detecting magnetic field in main-sequence stars

The magnetic field of CP stars was detected much earlier than in other objects because of its large value and the simplicity of its geometrical structure. Nevertheless, an interpretation of the results obtained in observations is not a simple problem.

The transfer equation for the Stokes vector and its solution are applicable to a local point on the star surface. We record the emission integrated over its entire visible surface. Thus, we observe the magnetic field averaged over the star surface. That is why the methods of spectropolarimetry are effective in the case of large-scale magnetic fields of global character. If the star has a lot of regions with fields of different polarity, then the method efficiency degrades because: 1) the contributions of the star surface regions with opposite fields are compensated and so only the residual fields are measured; 2) the contribution of emission from the nonmagnetic regions considerably reduces the measured intensity.

Practically all measurements of magnetic fields of CP stars are based on the study of the Zeeman splitting of spectral lines. The measuring of longitudinal components of a large-scale stellar magnetic field of global character is the easiest. To be detected, the field must be coherent, i.e. one prevailing polarity must appear as a result of superposition of emissions from different regions of the stellar surface (Babckok 1967).

If the star is oriented in such a way that we basically observe the longitudinal field, then to reveal the relative shift of lines with left or right rotation of the electric vector, an analyzer of circular polarization can be used. If the star field is coherent and mainly transversal, then using the differential analyzer for two directions of linear polarization differing by 90° in azimuth, we obtain two unbiased lines of different sharpness. Since it is much easier to determine the shift than the difference in the profile sharpness, the basic technique is the measurement of the longitudinal Zeeman effect with the analyzer of circular polarization. This technique was suggested and realized by Babcock (1947 a, b). It consists in measuring the Zeeman shifts of opposite polarized σ-components of separate lines.

Magnetic CP stars have large-scale magnetic fields and can be studied with spectropolarimetry techniques, but since in any case a magnetic broadening of lines occurs, the non-polarimetry techniques can also be applied. However the precision of such determinations is much worse (the limit of detection is about 2 kG for narrow and sharp lines instead of 200–300 G obtained with the spectropolarimetry techniques).

Nevertheless, since only a very small part of spectra was obtained with the Zeeman analyzers, the elaboration of techniques that permit diagnosing magnetic fields on the ground of already available usual (non-polarized) high-resolution spectra seems to be very useful. Its implementation will allow using large archives of observational data in the study of stellar magnetism.

4.2. Non-polarimetric measurements of magnetic fields

These methods were first used to search for magnetic fields of complex structure in stars for which there were indirect indications to the presence of field, but the classical measurements by Babcock's method failed.

Robinson (1980) put forward the following method: profile comparison of lines with similar spectral features, but with the substantially different Lande factors. The following parameters are compared:

1) the normalized depth d (in units of continuum intensity);

2) the chord length at a level zd ($z = 0.3, 0.5, 0.7$), in terms of km/s;

3) the area below the chord at the half depth; this value is less susceptible to the blending effect.

Along with the absolute field value, this method also allows determination of a part of the star hemisphere covered with fields. In most cases, however, only fields stronger than 1 kG can be detected with this technique.

The main factor influencing the result is the blending, which is especially strong for cool stars. Therefore, pure lines should be used if possible. There are other limitations: 1) it is more preferable to take weak lines in the linear part of the curve of growth; 2) a precise configuration of the Zeeman pattern should be known; in most cases the modeling by a simple triplet is impossible; 3) different components have different intensities and, consequently, saturations at the curve of growth, therefore, their contribution into the common profile depends on intensity.

Following the pioneer paper by Robinson (1980) and using the same basic principles, astronomers detected more than 50 late dwarfs with magnetic fields. For cool stars (by analogy with the Sun) a two-component model is accepted, in which there are magnetic and non-magnetic regions. However, there are essential differences.

Magnetic fields on the Sun reach values of kilogauss in structures of two types: in solar spots of several thousand kilometers in diameter which are cold and dark in the visible range, and in active regions where the fields are concentrated in small flux tubes more bright and hot than the surrounding photosphere, occupying less than $^1/_4$ of the active region area. The field reaches 1–2 kG there, whereas in spots it reaches 5 kG. The tubes together take less than 1% of the solar surface, and on 99% of the area the field is weaker than tens of gauss.

In cool stars there are magnetic and non-magnetic regions, a very narrow field intensity distribution (1–2 kilogauss) being observed. In stars the filling factor (the ratio of the area of the region with a field to the total area of the visible surface) is $\geq$15 %, which can mean that the fields are not isolated in small tubes as in the Sun. The study of magnetic fields of cool stars is a new rapidly developing branch of investigations. There are many publications analyzed in the recent review by Valenti and Johns-Krull (2001).

The study of global fields of CP stars by magnetic broadening will be discussed below. Such investigations have not gained wide recognition because of a substantially lower accuracy and less informativeness than obtained from analysing spectropolarimetry data.

4.3. Study of longitudinal magnetic fields by measuring circular polarization in lines

4.3.1. Photographic technique

The information about magnetic field is obtained from measuring the shift of left circularly polarized (LCP) and right circularly polarized (RCP) lines. In case of a simple triplet the centers of lines observed in the RCP light coincide with the positions of ($\sigma+$)-components, and for those observed in the LCP light with the positions of ($\sigma-$)-components. In the case of the anomalous Zeeman effect the shift $\lambda_r - \lambda_l$ is strictly proportional to B_e, but the effective Lande factor must be replaced by the number depending on intensity for precise reconstruction of all σ-components because of the different saturation of each individual component in the pattern of splitting.

The value of the longitudinal magnetic field B_e can be determined from the formula suggested by Babcock (1958):

$$\Delta\lambda = \pm 4.67\cdot 10^{-13} z B_e \lambda^2, \qquad (9)$$

where B_e is the longitudinal field component, z is the Lande factor of the line, $\Delta\lambda$ is the shift of each separate line in opposite polarized spectra (in Å).

The detailed description of photographic techniques is caused not only by a historical interest. First, about half of available Zeeman spectra were obtained on photographic plates. The comparison of new and old data is important, especially when studying the long-period variability of magnetic CP stars. Second, though this technique is called a classical photographic one, these name and technique are also applied to the case where observations were carried out with modern CCDs. The measurements of magnetic fields obtained with this technique for the case of CP stars with simple dipole fields are interpreted unambiguously enough. After introduction of digital registration of spectra, many disadvantages of early photographic method related to non-linear characteristic of the photoemulsion curve were eliminated. Besides, the degree of automation increased considerably.

Returning to photographic plates, note that the precision of recording on them was low, which did not permit studying the profile details, but only some parameters describing the line on the whole (equivalent width, etc.) That is why, although some separate attempts to measure the transversal field were undertaken (Glagolevskij et al. 1985b; Kodaira and Unno 1969), this work received development only with introduction of new digital light detectors. The first positive results of Doppler mapping (e.g. Khokhlova 1983) were obtained with photographic plates, but their insufficient precision made the Zeeman mapping be impossible. It became possible only with the appearance of CCDs.

The accuracy of the line shift measurements drops sharply with increasing their width. Earlier it was practically impossible to photographically determine magnetic fields for stars with a line width of more than 0.6–0.7 Å. It was also necessary to take into account numerous distorting factors. Let us consider some of the most essential of them.

1. Sometimes the field values obtained by different researchers were considerably different. Hensberge (1978) studied systematic differences in B_e values measured at different observatories. The differences in the quality of polarization analyzers, seeing during observations, and also the methods of correction for instrumental polarization, most strongly affecting the results obtained with the coude spectrographs, turned out to be decisive. Usually a compensator of instrumental polarization was used. It was not used

at the 6 m telescope because at the Nasmyth focus the polarization is small and was taken into account mathematically (Glagolevskij et al. 1977).

2. In the days of Babcock the magnetic fields were estimated by visual determination of the line component centers of gravity in opposite polarizations on photographic plates, i.e. it was quite subjective. Each researcher had his own system and his own error. Later the oscilloscopic attachments were introduced, which allowed improvement of the accuracy and objectivity of measurements. Nevertheless, due to non-linearity of photoemulsion, the researcher saw the line core better, and the visible center of gravity shifted towards him. The magnitude of this shift was dependent on the line intensity.

The problems of blending are especially vital for cool CP stars. However, when measuring the longitudinal field component B_e the blending cannot introduce large errors and distortions since the Lande factors of most lines are in a rather narrow range of values (z = 0.9–1.6).When using many tens of lines the average Lande factor is within 1.20–1.25 irrespective of the spectral band. However, it was noted that the lines of different chemical elements differ in their sensitivity to the field. So, the analysis of our catalog (Romanyuk 1984) has shown that the mean Lande factors for different lines are: 1.07 for TiII, 1.17 for FeII, 1.32 for FeI, 1.23 for CrII. 1.25 for CrI, 1.82 for EuII, 1.88 for GdII. It can be seen that the europium and gadolinium lines have large factors, whereas for the rare-earth element cerium the situation is different: the average Lande factor of the line CeII is equal to 1.01. Thus, the errors in the field measurements will affect the value of the determined field only in the case of wrong identification of the rare-earth element lines.

Another circumstance seems to us to be more essential: due to the non-uniform distribution over surface, the spots of higher concentration of one or another element will be in different regions of the star surface with different fields, therefore, its value determined from metal lines can differ for different elements and degrees of ionization. Even in the case of a dipole field the values of B_e determined from different elements can differ. It was confirmed many times in direct measurements (for example, in the paper by Glagolevskij et al. 1984).

To obtain the Zeeman spectra the observers used different version of the Babcock analyzer. Babcock himself worked at the 5 meter telescope of the Palomar observatory and the 2.5 meter telescope of the Mount Wilson observatory. The technique is described in his papers (e.g., Babcock 1958). In his famous catalog ibidem the results of magnetic field measurement for 89 stars are presented.

Many observations were carried out with the 3 meter Lick telescope. The technique is described in the paper by Preston (1967). The most considerable results were obtained with this telescope by G. Preston and his colleagues S. Wolff and K.Stepien (e.g. Preston and Stepien 1968; Preston 1969a,b; 1970, 1971, 1972; Preston and Wolff 1970, and others). From Babcock's studies is was clear that magnetic fields of A type stars exist, they are strong and can be reliably measured. Preston and his colleagues found that the longitudinal field component varies periodically, synchronously with the variations of brightness in spectral lines. The period of variability is equal to the period of star rotation. Thus, an oblique rotator simple model well explaining many properties of magnetic CP stars was constructed. Preston first found splitted Zeeman components for several magnetic stars, which permitted him to determine not only the value of the longitudinal component, but also the so called "surface field" B_s — a mean modulus of the magnetic field averaged over the visible surface.

In the 70s and 80s V. Bonsack and S.Wolff actively measured magnetic fields with the 2.24 meter Hawaii telescope. The equipment and techniques of their observations are described in the paper by Wolff and Bonsack (1972). Wolff (1978) first tried to detect the radial gradient of magnetic field from lines produced in different atmosphere layers. But she was working with a mica analyzer and, therefore, could not achieve high precision. As a result, the conclusions of the Wolff's work are rather uncertain.

The photographic observations of magnetic fields were also fulfilled with the 2.7 meter telescope of the MacDonald observatory (Vogt et al. 1980), the 3.6 meter ESO telescope (Mathys and Stenflo 1986), the 2 meter telescope in Tautenburg (Scholz and Gerth 1981) and others. As regards the USSR territory, before the SAO a Zeeman analyzer was set on the 2 meter telescope in Shemakha (Aslanov and Rustamov 1975).

The magnetic observations with the classical Babcock analyzer started at the SAO 6 meter telescope in 1976 (Glagolevskij et al. 1977). Then the analyzer was updated (Najdenov and Chuntonov 1976): instead of mica plates the Fresnel rhombs were used as phase-shifting elements, which made it achromatic. The need for a set of analyzers for working in different spectral bands was eliminated. Besides, each of them had its own individual features that could not be taken into account completely.

The measurements of the Zeeman spectra were carried out with the help of oscilloscopic attachment by superposing the direct and reversed images of the spectral line evolvent (Antropov 1972). This measuring system was of much higher precision and objective than the visual one, but in the case of stars with complex line profiles some subjectivism remained and the results depended to a large extent on the researcher's experience.

The photographic observations of magnetic field had been made with the 6 meter telescope from 1977 to 1990, i.e. practically till the end of the photographic era in astronomy. During all that period more than 2000 Zeeman spectra were obtained, basically with the second camera of the Main Stellar Spectrograph.

In the 90s of the 20^{th} century they started using CCDs as detectors with practically all instruments (including the 6 meter telescope). The digital recording of spectra permitted rendering the process of measuring magnetic fields automatic to a great extent. The procedure of the Zeeman spectra analysis will be described separately. Here we only note that the observational material obtained with the 6 m telescope is presently being processed with the software by Kudryavtsev (2002). Observations of standard stars repeated many times showed that the replacement of photographic plates by CCDs and alterations in the processing did not lead to disruptions in the system of magnetic measurements at SAO RAS.

Concluding the description of photographic technique, note that since the Lande factors of most lines are not much different from each other, then, using a rather narrow spectral range (less than 500 Å) one may accept to a first approximation that the influence the Zeeman effect has on profiles of all measured lines is the same. This circumstance is attractive because presents a possibility to use a "cumulative" effect of many lines: the polarization signal will be accumulated much faster than noises, therefore the signal-to-noise ratio increases with increasing number of used lines. It was assumed that the development and application of such multi-line technique will be especially useful in search for weak magnetic fields and in the study of fast rotators.

4.3.2. Correlation technique MSHIFT

This technique for express analysis of photographic Zeeman spectra was advanced by Weiss et al. (1978) in the Vienna observatory. Its application was assumed to lead to automation and considerable speeding-up of the laborious process of magnetic field measuring.

The technique consists in searching for correlation between the reference and investigated spectrograms with the help of a digital micro-photometric system. In this technique the shift between a pair of differently polarized spectra is eliminated numerically. Thus, two spectra weighted in intensity are considered. The required shift $\Delta\lambda$ is a minimum of the sum of discrepancies of intensity differences of two orthogonally polarized spectra. It is interpreted in units of magnetic field:

$$\Delta\lambda = 2\langle z\rangle\langle\Delta\lambda\rangle B_e, \tag{10}$$

where $\langle z\rangle$ is the averaged value of the effective Lande factor in the Lorentz's units, $\langle\Delta\lambda\rangle$ is the mean line shift.

Weiss et al. (1978) and Mathys (1989) mention the following advantages and disadvantages of the technique.

Advantages:

- its objectivity (in the traditional method the determination of the line centers of gravity is made visually and, therefore, subjectively);
- the preliminary identification of lines is not necessary (however, it is necessary to find an adequate value of $\langle z\rangle$);
- all information about the spectrum is used.

Besides, an indubitable positive factor is a high degree of automation of the observation processing.

However, MSHIFT has a number of considerable disadvantages:

- the use of averaged values of $\langle z\rangle$ and $\langle\Delta\lambda\rangle$ leads to the fact that the real contribution of each line depends on its intensity;
- the stronger lines contribute more; there are difficulties in the case of the anomalous Zeeman effect (the curves of growth for components of different intensity are different);
- there is no direct relation between measured shifts and the magnetic field or radial velocity, which are required for empiric calibration;
- the false magnetic variability can arise when elements are distributed in the star atmosphere in a non-uniform way, which can generate spectral variability.

Actually this technique determines the mean Zeeman (or Doppler) shift of lines averaged in intensity.

A numerical simulation fulfilled by Stift (1986, 1987) showed that even at the uniform chemical abundance the application of MSHIFT leads to dependence of the obtained value on the velocity of rotation, therefore the obtained value can differ much from the true one. A conclusion was made that MSHIFT is suitable in searching for magnetic fields, but not in their measurement. Practically nobody except the authors of MSHIFT used this technique, basically because of difficulties in interpretation of obtained results. Nevertheless, we decided to describe it here, since MSHIFT is the first correlation technique applied in magnetic measurements.

4.3.3. Photoelectric technique: differential and integral methods of field measurements

Photoelectric stellar magnetometers were first used in the late 60s by A.B. Severny in the Crimean observatory (Severny 1970) and by a group of Canadian astronomers (Borra and Landstreet 1973).

Briefly, the technique essence is as follows: the light accumulated by a telescope passes through an electro-optical modulator, the voltage of which plays a role of a quarter-wave plate. At periodic alteration of the voltage sign, the ordinary and extraordinary axes of the modulator alter. Then the beam goes through the analyzer that transmits the linearly polarized light along a given direction and entirely blocks the perpendicularly polarized light. The polarization direction in the analyzer must be at an angle of 45° with respect to the modulator axes. Then the beam enters the spectrograph, at the exit of which there is a slit to select a narrow spectral band recorded by a photomultiplier. The difference between signals from two (RCP and LCP) polarized spectra gives the Stokes parameter V in the selected spectral band, and their sum gives the Stokes parameter I in the same element. The exit slit can move along the spectrum.

In the first photoelectric devices the spectrograph exit slit was wide enough (0.2 Å) (Severny et al. 1974). It led to a high signal-to-noise ratio, but the resolution became worse. Later, to improve the efficiency the Fabry-Perot interferometer was set in front of the slit (see Borra and Vaughan 1977; Glagolevskij et al. 1979). Such a device was also built for the 6 m telescope. Its description can be found in papers by Glagolevskij et al. (1979), Bychkov et al. (1988). The employment of the interferometer made the observational process complicated, but the light transmission increased. The magnetometer turned out to be much more effective than the devices of the same spectral resolution without it.

But this direction did not evolve in our country or abroad. The magnetometers with interferometers were used only for the unique one-time tasks (e.g., for measuring magnetic fields of the brightest stars accurate to order 1 G (Borra 1975), searching for weak magnetic fields (for example, Glagolevskij et al. 1989), for observing the circular polarization with high spectral resolution inside lines (Borra 1980 a, b). The main cause of so limited application is the single channel.

Since the photoelectric technique measures the circular polarization in wings of metal lines, the interpretation of results is more complicated than when measuring the line shifts. In addition, the more real field value can be obtained only in the case of uniform distribution of elements over the surface.

And what is more essential, the fields must be weak, and the broadening by rotation must be small. The typical half-width of metal lines of Ap stars is several hundredths of Å, i.e. one have to choose objects with v sin i $<$ 1 km/s and the magnetic field less than 1 kG.

The magnetic field value can be determined in two ways: a differential method (Borra and Landstreet 1973) and an integral one (Borra et al. 1973).

In the differential method one measures the polarization in the line wing, determines the profile slope dI/dλ and, on that ground, calculates the value of B_e by the formula

$$B_e = 2.14\cdot10^{12}(V_\lambda/\lambda^2)/z\frac{dI}{d\lambda}, \tag{11}$$

where V_λ is the Stokes parameter obtained in the given profile point, dI/dλ is the profile slope in this point.

In the differential method the magnetic field is determined as the shift of the centers of gravity in the distribution of the Stokes parameter V in the left and right wings, which is somewhat closer to the methods of classical photographic observations. The value of the longitudinal component of the magnetic field is

$$B_e = 2.14\cdot10^{12}(\Delta\lambda/\lambda^2)/z, \tag{12}$$

where $\Delta\lambda = \int\lambda V_\lambda d\lambda\cdot\int R_\lambda d\lambda$, R_λ is the line depth as a function of wavelength.

The above-mentioned papers by Borra and his coauthors demonstrated that the difference in the field values determined by the two methods can reach 30%.

Recall that even Severny et al. (1974) noted that the field determined by the outer and inner line wings can differ because of the influence the π-component has on the inner wing. The pure longitudinal field can be determined by the distribution of circular polarization in the outer wing, which is free from this influence.

We think that the values obtained by the integral method were more reliable and stable, closer to classical photographic measurements. Nevertheless, the difference between the field values obtained from metal lines with photoelectric and photographic techniques is so large that the common usage of these data in search for long-term field variations does not seem to be possible.

Let us itemize the main sources of errors in the field determination with the magnetometer equipped with the Fabry-Perot interferometer.

1. The single-channel character of the device requires scanning of the profile, which abruptly reduces the efficiency. To save time the measurements were

fulfilled only in selected points, therefore the field value can be distorted.

2. The necessity for very careful adjustment of the polarization optics and monitoring the modulation depth.

3. The air system of the interferometer is complicated and uncertain. It is impossible to reproduce exactly the necessary wavelength.

4. The necessity for careful account (to an order of 0.01%) for the instrumental circular polarization which is equal to 1–2% at the BTA Nasmyth focus.

As a result, the magnetometers of that kind were used during a relatively short period of time. Their awkwardness and the necessity for long many-hour observations to obtain one measurement made them noncompetitive for the work with the big telescopes.

The observations with magnetometers operating in the wings of the hydrogen lines turned out to be more effective. Such a device was first demonstrated in the paper by Angel and Landstreet (1970). It was meant to measure magnetic fields of white dwarfs and fast rotating CP stars. Note that the world's second hydrogen magnetometer was built for the 6 m telescope by V.G. Shtol' (Shtol' et al. 1985).

The hydrogen lines have a number of advantages in comparison with the metal lines: hydrogen is an element which is distributed the most uniformly over the star. They are broad and, consequently, only slightly susceptible to the broadening by rotation. A good method to measure magnetic fields of rapidly rotating stars (with $v \sin i > 200$ km/s) appeared. Because of the large width of the hydrogen lines, high spectral resolution is not necessary which leads to a considerable simplifying of equipment and makes observations more effective.

The profile slope in the hydrogen line wings is much lower than in the metal lines. Therefore, to achieve high accuracy, it is necessary to provide a large signal-to-noise ratio. Thus, when using the spectral band of width 5 Å in the H_β line wing for a typical CP star 1% of the recorded polarization (the Stokes parameter V) corresponds to a magnetic field of about 13 kG. It follows that at the fail-free operation of equipment and after accumulating 10^8 photocounts we obtain the accuracy of polarization determination of 0.01% and that of magnetic field of $B_e = 130$ G.

Some difficulties emerge when using this technique.

1. The profile slope $dI/d\lambda$ is not known exactly. It cannot be determined from observations only in the wings. This requires independent observations or the modeling of line profiles, otherwise inaccuracies can arise when scaling the magnetic measurements.

2. The signal of the Stokes parameter V is interpreted in terms of B_e as the weak field approximation in the Milne-Eddington solution of transfer equations. In the case of hydrogen lines the strong Stark effect takes place and the proportionality between I and $dI/d\lambda$ can be broken. It means that the different parts of the hydrogen line profile can give different field values even if the effects from broadening by rotation and magnetic field are small.

Being the basis of the Milne-Eddington solution, the LTE hypothesis is to be accepted. The hydrogen lines of A and B stars cannot be always described well. The combination of the Stark and Zeeman effects complicates the issue.

Nevertheless, an experience shows that the longitudinal magnetic fields obtained from the polarimetry of Balmer lines fit well. However, the discrepancies between curves of the longitudinal component obtained with photoelectric and photographic methods occur often. It is exhibited in different sharpnesses of curves of B_e variability with the rotation period phase (e.g. Borra and Landstreet 1980).

In particular, Borra (1974a) considers that the field determination by the photographic method is wrong. He calculated theoretical line profiles on the assumption of the oblique rotator model and concluded that the contribution of the line cores in photographic determinations is too large, which can cause a false increase in the field up to 2 times in comparison to the real value. But many researchers do not agree with this assertion because there are other reasons of discrepancies, the non-uniform distribution of metals over the star surface being the main of them.

The details of the SAO hydrogen magnetometer structure are described in the paper by Stol' et al. (1985). We only note here that unlike Landstreet's device (Angel and Landstreet 1970) it is a universal spectropolarimeter in which the necessary band is selected by a set of mask-slits. The use of the spectrograph instead of the interferential filter made the structure somewhat more complicated, but it permitted the device to be used for observations of magnetic fields and polarizations (at first, the circular polarizations, and then the linear polarizations) for a wide range of objects. It was used to measure magnetic fields of many CP stars and to search the field for stars of other types (e.g., λ Boo (Iliev et al. 1990), Ae/Be Herbig), to measure the 4 Stokes parameters (Glagolevskij et al. 1988, 1990; Kudryavtsev et al. 2000).

The device was reliably gauged and operated stably. It was excluded from the list of standard devices only in 1995, when its exploitation was already impossible because of deterioration and lack of spares.

4.3.4. Multi-line Zeeman polarimetry

To increase the efficiency of single-channel photoelectric polarimetry there was an idea (by analogy with MSHIFT) to observe many lines simultaneously by replacing one slit to a mask with many slits imitating the stellar spectra, as was made in CORAVEL (Baranne 1979).

Brown and Landstreet (1981) described the equipment and technique of multi-line Zeeman observations. The operation with such a device looks like this: at first, the mask is positioned till the coincidence with the centers of stellar lines (for that the standard mode is used — the measurements of radial velocities). Then the Zeeman analyzer is set in and the mask is shifted a little. At that, the circular polarization from wings of many lines is recorded in the same way as for one line. The only difference is that the signal is averaged over a large number of lines.

The limitations of this technique are the same as for the MSHIFT method. Namely, 1) the interpretation is possible only for a small Doppler and Zeeman broadening; 2) the interpretation is possible only for the lines that are included in the mask and for an element with a uniform distribution over the star disk; 3) the mean Lande factors and shifts $\Delta\lambda$ are susceptible to the same distortions as for MSHIFT.

Thus, the multi-line analysis with the device of the type of CORAVEL does not fit well for obtaining more real observations, which was confirmed in numerical modeling by Stift (1986).

But this technique is effective when looking for magnetic fields because of its very high sensitivity: for example, the fields of several gauss were detected with it (Borra et al. 1984). Later on, when multichannel digital light detectors appeared, the optical masks were replaced by mathematical ones.

4.4. LSD-technique for the study of magnetic fields

The correlation technique of magnetic field measurement known as LSD (Least-Square Deconvolution) was developed in the papers by Donati et al. (Donati and Collier Cameron 1997; Donati et al. 1997). At present it is widely used in the study of magnetic fields of cool stars. The technique essence is addition of a useful signal from many lines with similar behaviors in magnetic field.

The construction of the spectropolarimeter MUSICOS by Donati et al. (1999) at the Pick du Midi Observatory was a big achievement in development of observational facilities of magnetometry.

It is used to observe IQUV spectra (i.e. all four Stokes parameters) in a band-width of more than 2000 Å with resolution 35000. To process these spectra, new correlation methods were suggested.

4.4.1. Correlation techniques of measuring circular polarization

The problem of obtaining the Stokes parameter V signal with a high signal-to-noise ratio can be solved by summarizing the shifts of many lines in magnetic field in such a way that the shifts are directed to one side. Unlike MSHIFT, the mathematic masks are used; therefore it is possible to work with arbitrary lines.

It is assumed that the forms of the V parameter profiles are the same, and the signal depends on the product zd (where z is the effective Lande factor of the line, d is its depth). Therefore the signals from many lines can be added. The paper by Donati and Collier Cameron (1997) can exemplify the search for magnetic field of cool stars with this technique.

The LSD technique can also be applied to CP stars, where the weak field approximation is not a quite good approach (Wade et al. 2000b). It turned out that the V profiles can be widely used for the magnetic field measurements of CP stars: the field moment $\langle B_z \rangle$ calculated by LSD observations changes smoothly with the period phase; the spread in values is very small. Though these results agree well with data obtained by other techniques, their interpretation is not simple.

The paper by Shorlin et al. (2000a) is an example of high-precision measurements of the longitudinal field component for CP stars. With the help of the spectropolarimeter MUSICOS the authors obtained observations of the longitudinal component of magnetic fields for 23 Am and Hg-Mn stars accurate to 30–40 G, which exceeds the precision of previous results by one order. The field was not detected.

4.4.2. Correlation techniques for measuring linear polarization

The LSD technique is more useful in the study of linear polarization in lines. It was first used when searching for weak polarization with the eshelle spectrograph MUSICOS. Its application permitted reliable detection of weak linear QU polarization for a number of magnetic Ap stars, which was not detected in studies from separate lines.

The LSD technique is a cross-correlation procedure meant especially for extraction of a weak signal (of order 0.01%–1% from the total amplitude). In the case of search for linear polarization the technique advantage is that the profile shape (and related polarization features) changes slightly from line to line

(Wade 2002).

The survey with MUSICOS permitted proper linear polarization in lines for more than 10 Ap stars to be detected. For some of them the proper broadband linear polarization was not detected because of the large interstellar linear polarization, or the small line density, or because of saturation (Wade et al. 2000a).

In the cases where both data of broadband polarimetry (Leroy 1995) and MUSICOS polarimetry data in lines (Wade et al. 2000a) are available, they agree well, which confirms once more that the broadband polarization is caused by saturated spectral lines (Wade et al. 2000b).

However, even in case of the simplest dipole field the predicted QU profiles of separate lines differ considerably from each other. Although the LSD technique allows detecting linear polarization, the relation between the LSD profile and the profiles of linear polarization of separate lines is far from being clear: the latter are very sensitive to the value of $v \sin i$ and the field configuration.

4.5. Determination of the module of the mean field

The magnetic field affects the lines in such a way that the Stokes parameter V is not equal to zero, and, besides, it also distorts the parameter I, i.e. the spectral line profile. This distortion is seen for stars of solar type and is even more pronounced for CP stars where the filling factor is equal or very close to 1. The best display of magnetic fields in the intensity spectra is the presence of lines which are clearly splitted into the Zeeman components, like for HD 215441. In less favorable cases the field becomes apparent in the broadening of lines. The diagnostic of magnetic fields from unresolved profiles for Ap stars was first fulfilled by Preston (1971).

4.5.1. Lines with resolved components of the Zeeman splitting

In some CP stars the spectral lines are splitted into components. They are observable only under certain conditions: 1) the non-magnetic width of spectral line must be smaller than the Zeeman splitting; 2) the field must be substantially uniform over the visible stellar surface, the field variation would cause line broadening; 3) the Zeeman pattern of splitting must be simple, with a small number of components.

The first condition is realized if the field is large enough and the star rotates slowly or is viwed from the rotation pole. Due to this, the number of stars with observable splitting is small. The second condition describes the Ap stars on the whole. All objects with observed split components have narrow lines. This is an indication that their fields are uniform indeed. Besides, they cover all or almost all stellar surface, as is shown, e.g., in the paper by Mathys et al. (1997): the pattern of splitting of the line FeII 6149.246 into two components in the spectrum of the star HD 94660 shows that the intensity between the components reaches continuum (there is no central component). The third condition is also intelligible: the lines with the simple splitting and large Lande factors are the best for usage.

Thus, the measurements of relative shifts with the Zeeman pattern in the form of a doublet or a triplet can by interpreted in terms of the ($\sigma\pm$)-component weighted in intensity and averaged over the star disk. The result does not depend on the field vector orientation. If we find relative intensities of resolved ($\pi-$)- and ($\sigma-$)-components, then we could estimate the field pattern.

4.5.2. Lines broadened by the Zeeman effect

The line splitting is rarely seen; in most cases a differential broadening of the Zeeman components occurs. Let us consider what information can be extracted in this case.

Preston (1971) was the first who used the differential broadening for quantitative study of magnetic fields. He measured the full width W for a sample of lines with different sensitivities to magnetic field and assumed that it depends on the effective Lande factor z and on the wavelength in the following way:

$$W^2 = W_0^2 + K^2 \lambda_0^4 z^2, \tag{13}$$

where W_0^2 is the proper line width in the absence of field, including the equipment profile and the Doppler broadening, K is a coefficient proportional to the magnetic field value.

The values of W^2 for lines with the large and small Lande factors are averaged separately. However, this approach leads to substantial difficulties. The most fundamental one is as follows. Generally, the width of a completely split Zeeman line does not depend on the field geometry, whereas for the incompletely split one it does. It is subconsciously comprehensible by the example of a simple triplet. When such a line is splitted completely, its full width is determined only by the wavelength splitting and the width of its σ-components. When the line is splitted incompletely, the π-component contributes to the line width according to the field geometry.

Didelon (1987) attempted to apply Robinson's technique to the diagnostics of magnetic field of Ap stars. He compared the mean magnetic field module obtained from observations of resolved lines to one obtained by Robinson's technique. A difference was found between the field values obtained from two pairs of lines. Its cause is still to be ascertained. Probably, the problems are related to the technique employed in observations of cool stars.

Due to the large-scale structure of magnetic field, its action in a point of the stellar surface correlates with the Doppler shift of radiation emitted from this point. This shows that Robinson's technique can be used only for stars with a negligible value of v sin i. Similarly, the angle γ between the line of sight and the local magnetic vector also correlates with local field value. That is why the line width variability can be greater in the magnetic regions of solar-type stars where the widths depend only on γ.

All diagnostic techniques based on the study of the Stokes parameter I have several limitations, the main of which is that they can be used only for stars with very slow rotation.

4.5.3. Magnetic intensification of lines

The magnetic intensification is the increase of the equivalent width according to the splitting character, the magnetic field value and direction, the parameters of absorbing medium. It unambiguously depends on the product of the number of components and the splitting increment $n\delta$ (Boyarchuk et al. 1960).

Since the magnetic field broadens the profiles of absorption lines, the increase of equivalent width proportional to the field value arises because of saturation and non-linearity of the curve of growth, which was first noted by Unno (1956). This effect should be taken into account when studying the element abundances in CP stars.

I showed (Romanyuk 1984) that the anomalous intensifying of rare-earth lines in the spectra of CP stars can be explained by the magnetic intensifying provided that the field value is equal to 20–25 kG on average, but it contradicts observations.

Preston (1970) studied the magnetic intensifying of lines in the spectrum of HD 126515. The surface field of this star is from 10 to 17 kG, that allows us to observe its manifestations. The field action was conceived as pseudo-microturbulence, and its value was obtained simultaneously with the precise analysis of chemical abundance.

This technique gives a coarse estimate of the field module in the case where more precise methods cannot be applied. This technique was first used by Hensberge and De Loore (1974). The average magnetic field modules of several stars were found by Ryabchikova et al. (1984; 1988), the accuracy of obtained estimates being low, of order 1 kG or worse. Kolev (1977) showed that the method of comparison of observed and theoretic relations between W_λ and lg gf for the lines of one and the same super multiplet allows revealing the magnetic intensifying of lines. A method for estimating the field value from the relative intensification of lines with different z and close values of lg gf was suggested.

4.6. Mathys' method of moments

In the 80s and 90s, a famous explorer of magnetic stars G. Mathys published a number of papers in which the method of moments for determining the magnetic field parameters was developed. The method principles are presented the most logically in the paper by Mathys (1989). Below there is a brief description of the method essence according to the papers by Mathys (1989, 2001).

The features of a spectral line in magnetic field are determined by the transfer of polarized emission. Consider 4 transfer equations for 4 Stokes parameters I, Q, U, V in the Milne-Eddington approximation of the atmosphere (the linear function of the source, the ratio of the absorption to continuum is constant). In the case of weak lines and weak field the solutions are simple.

The assumptions used in most methods of magnetic field measurements are as follows:

1) the average separation of the centers of circularly polarized $(I+V)$ and $(I-V)$ lines is proportional to $z\langle B_z\rangle$, where z is the effective Lande factor, and $< B_z >$ is the magnetic field component averaged over the line of sight;

2)in a weak field approximation (and at a small v sin i) $V \sim z\ Bz\ dI/d\lambda$ in the local line;

3) the separation of resolved σ-components is proportional to $z\langle B_z\rangle$.

The integration is made over the whole observed line. The limit of integration can be far in wings: to a maximum possible Zeeman shift of line in combination with the Doppler effect in any point of the stellar surface.

Especially interesting is the second moment which is a dependence of the profile shape on the square of the mean magnetic field and its component along the line of sight averaged over the visible stellar disk and weighted by the intensity in line. If the profile is determined in an independent way (e.g., from theoretical models or by observations of zero lines, as in Robinson's method) the magnetic contribution to the second moment can be separated. In principle, the contribution of $\langle B^2\rangle$ and $\langle B_z^2\rangle$ can be distinguished with the help of two lines with different patterns of the Zeeman splitting.

The modeling of a field obtained in such measurements is traditionally reduced to calculations of expected values of $\langle B_z \rangle$ and $\langle B \rangle$ for a multi-pole model of magnetic field and the comparison of them with observed moments. Since the behavior of most values of $\langle B \rangle$ and $\langle B_z \rangle$ is close to a sinusoid, then the model of the dipole magnetic field for CP stars have strong reasons. But even for this simplest model three parameters should be known (the inclination i of the rotation axis to the line of sight, the inclination β of the magnetic axis to the rotation axis, and the field value B_d at the pole), i.e. the single model cannot be obtained without an independent determination of i.

4.7. Indirect methods

Consider below two methods that allow detecting some indications of the presence of stellar magnetic fields. But they do not allow measuring the field value.

4.7.1. Magnetic fields obtained with the Geneva photometry

Continuous spectra of CP stars show broad (200–300 Å) and shallow (of depth of several percent of intensity) depressions. These anomalies in the energy distribution, which are typical of CP stars, were detected by Glagolevskij (1966) and Kodaira (1969). They developed different criteria that allow distinguishing between normal and CP stars on the ground of midband photometric observations fulfilled, in particular, in the region of the strongest depression at 5200 Å (Straizys 1977; Maitzen 1976; and others).

Cramer and Maeder (1980), North (1980, 1984) found some proofs of correlation between the peculiarity parameters $\Delta(V_1 - G)$ and z of the Geneva photometric system on the one hand and the average magnetic field B_s determined basically from a paper by Preston (1971) on the other hand. This correlation exists only for the stars hotter than A5, since the parameter z cannot be actually determined for cooler stars.

Besides, Cramer and Maeder (1980) found an effect of saturation: for stars with B_s stronger than 5 kG the magnetic field and indicated parameters of the Geneva system do not depend on each other. They proposed a list of 258 stars with the surface magnetic field B_s stronger than 1 kG predicted from data of the Geneva photometry.

Unlike the magnetic field measurements, the photometry can be carried out with small telescopes, that is why in the case of operability of the indicated criterion it would be possible to abruptly increase the number of observations and to reveal new magnetic stars more effectively.

At the beginning of the 80s an extensive observational program was fulfilled with the 6 m telescope. The results were published in the papers by Glagolevskij et al. (1982, 1985b). 20 years later the investigation was continued but with the use of CCDs (Elkin et al. (2002, 2003) and others). These papers showed that there is indeed some correlation between the magnetic field value on the star surface and anomalies in the energy distribution in the continuum, but the dispersion of points in the required relation is too large.

During 25 years of observations we have detected more than 50 new magnetic stars, which is about half of all detected in the world in that period. Thus, the technique proved its efficiency in selection of candidates for magnetic stars: the anomalies of the continuum can be used as the indicators of field. Yet, the direct measurements should not be replaced by the results of indirect photometric determinations established by the empirical relationship.

4.7.2. Broadband measurements of linear polarization

This is one of the few techniques that can give information about the transversal magnetic field. The broadband linear polarization arises due to saturation of lines in a strong magnetic field: since the saturations of the π- and δ-components are not identical residual linear polarization arises. Leroy (1962) was the first who noted that. He also mentions (Leroy 1997) that a simple canonical model based on an oblique rotator model suggests some specific features: single- or two-twisted diagrams in the Q–U plane, which agree well with observational data. It was established that if both the longitudinal field component and the broadband linear polarization variability are known, it is possible to independently determine the angles i and β, typical of the dipole model of field.

Leroy (1995) made a big review of broadband linear polarization of CP stars with a specially built polarimeter. Proper variable linear polarization was detected for about 15 stars, and the parameters of the magnetic model were determined for them. For another 30 stars weak proper linear polarization or strong interstellar polarization was detected. In both cases no time variability was detected. Another goodness of this technique is that observations can be carried out with an instrument of moderate size (in the paper by Leroy (1995), this is the 2 m Bernard Lio telescope), which permits one to have a sufficient amount of observational time.

The results of the broadband linear polarimetry unambiguously point to the validity of the oblique rotator model for CP stars.

4.8. Technique of the Doppler-Zeeman mapping

To determine the magnetic field configuration on the star surface and its relation with the abundance anomalies, these values must be known in separate surface points. This is achieved by the technique of the Doppler-Zeeman (D-Z) mapping, which is, mathematically speaking, an incorrect problem.

Pioneer papers in this direction belong to V.L.Khokhlova and her colleagues. In her review (Khokhlova 1983) she wrote that the first attempt of a strictly qualitative approach to solution of the inverse problem — the obtaining of a map of a chemical element distribution over the non-uniform surface of a rotating star by observed spectral changes — was undertaken by Deutsch (1957, 1970). The method turned out to be inefficient because the integral line characteristics were used as input information.

Khokhlova proposed a mathematical model of solution of the inverse problem with the help of the local line profiles. Initially, the method was developed and applied to mapping chemical elements without taking account of the surface magnetic field and it was applied to the stars with a weak field (Khokhlova (1976); Wehlau et al. (1982); Wehlau et al. (1991), etc.).

On the other hand, software was developed, which was used for determining magnetic field without simultaneous determination or allowance for the "chemical map" (see, e.g., Piskunov (1985), Donati et al. (1989)).

Actually, on reaching some limiting field value and at marked chemical non-uniformity both problems should be solved simultaneously. It considerably complicates the solution of the inverse problem and presents substantial computational difficulties. Vasil'chenko et al. (1996) developed an effective method of simultaneous solution of equations for the Stokes parameters I and V, which makes it possible to obtain simultaneously the chemical element distribution and parameters of an arbitrary decentred magnetic dipole by the observed I and V profiles. However, the modeling of magnetic field involves some problems because the polarized line profiles depend not only on the value, but also on the orientation of magnetic field.

N.E. Piskunov developed a new direction in the mapping of magnetic stars. He and O. Kochukhov produced software that permits mapping the magnetic field vector and one more additional scalar parameter, e.g., the element abundance. The papers by Kochukhov (2000), Kochukhov et al. (2001), Piskunov (2001) describe the codes for the vector reconstruction of magnetic field over the star surface without any preliminary assumptions on its structure with or without allowance made for chemical composition non-uniformities. This problem has been discussed in details in a recent extensive investigation by Kochukhov (2004).

Landstreet (1988, 1990), Landstreet et al. (1989) obtained some results when solving the direct problem by the modeling in which the chemical map and the magnetic field configuration were tried simultaneously. Evidently, this method is extremely labor-consuming and rather subjective.

Bagnulo (2001) put forward his method to reconstruct magnetic field of peculiar stars. It consists in a direct modeling of the Stokes profiles on the basis of a priori assumption that the magnetic fields are potential and described by a limited set of free parameters. The application of this technique to specific stars is illustrated, e.g., in the paper by Bagnulo and Wade (2001). Gerth and Glagolevskij (2001) proposed another method to model stellar magnetic fields on the basis of distribution of "magnetic charges".

5. Conclusion

It is seen from the aforesaid that a lot of techniques were developed for analysis of magnetic field of CP stars. As a rule, one or another manifestation of the Zeeman effect in spectral lines is studied, which permits the necessary information about magnetic field to be obtained. The progress of this direction consists in both the increasing of observational accuracy and application of more and more refined modeling methods with taking account of the complex structure of real stellar atmospheres.

Presumably, a considerable advancement in the study of the magnetism of CP stars is to be expected in the near future. The expected breakthrough is related with implementation of new technology: magnetic observations started at the 8 meter ESO telescopes in Chile. The instrumental base of the 6 m SAO telescope has been considerably improved, which extends our possibilities. New method of the analysis of observations was also developed. The methods of Doppler-Zeeman mapping developed recently (e.g., by Piskunov) permits mapping the distribution of chemical elements and magnetic field without any preliminary assumptions. It is necessary to obtain high-quality observational material for as many stars as possible: I, Q, U, V spectra with high resolution and signal-to-noise ratio, with good coverage over the period phase for each star. The realization of such a program has been started at the SAO 6 m telescope. It should be expected that after its completion we will be able to understand important details of physics and processes occurring in atmospheres of stars with magnetic field and specific nature of energy release.

In the longer term, with development of interferometric observational methods in the optical band we can expect spatial resolution of disks of magnetic CP stars and direct observation of spots of chemical composition distribution and magnetic field configuration. But for this to be implemented, interferometers with a base not less than 1 km are required.

The equipment problems go beyond the scope of this paper. We deemed it expedient to consider them separately.

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